UNIT-I

INTRODUCTION to FLUID MECHANICS-I

Learning objective

At the end of this topic, you will able to

→ Explain dimensions & units of fluid mechanics.

→ Describe the physical properties of fluids

a) Specific gravity
b) specific weight
c) Density
d) specific Volume.

→ Define Viscosity, kinematic viscosity, Newton's Law of Viscosity, variation of Viscosity with temperature.

→ Describe types of fluid.

Outcomes

By the end of this topic, you will able to know,

→ Understand the concept of Dimensions and units of Fluid Mechanics

→ know the Physical properties of fluids and Viscosity, kinematic Viscosity, Newton's Law of Viscosity, Variation of Viscosity with Temperature.

→ Understand the Types of fluid.

Fluid Mechanics

→ The science that deals with the behavior of fluid at rest (fluid statics) & in motion (fluid dynamics), and the interaction of fluids with solids & other fluids at the boundaries.

→ Fluid - A substance in the liquid & gas phase

Fundamentals

Mechanics:

It is a physical science deals with both stationary & moving bodies under the influence of forces.

Statics:

Mechanics deals with bodies at rest.

Dynamics:

Mechanics deals with bodies in motion.

⟹ Definition of a Fluid:

* A Fluid is defined as a substance that deforms continuously under the action of a shear stress, however small magnitude present.

* It means that a fluid deforms under very small shear stress but a solid may not deform under that magnitude of the shear stress.

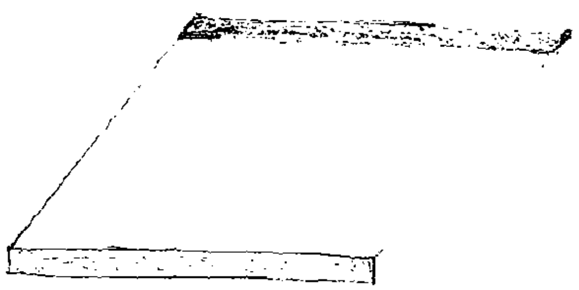


Fig 1(a) Deformation of solid under a constant shear force

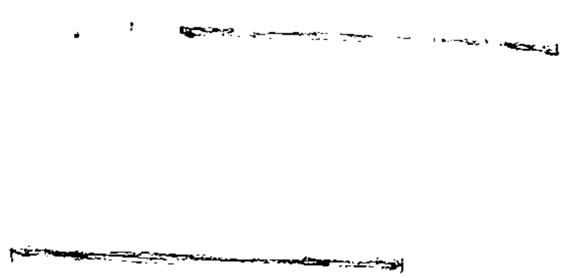


Fig 1(b) Deformation of fluid under a constant shear force.

* By Contrast a solid deforms when a constant shear stress is applied, but its deformation does not continue with increasing time.

* In the shown figure 1, deformation pattern of a solid & a fluid under the action of constant shear force is illustrated

Dimenssions and Units

Types:

→ Fundamental Units

All physical quantities are expressed in terms of Length Mass (M), Time (T).

→ Derived Units.

It is expressed in terms of Fundamental units

Area (A), Velocity (V), Acceleration (Ac), Pressure (Pr).

System of Units.

	Length	Mass	Time
CGS	cm	gram	second
FPS	feet	Pound	second
MKS	meter	kg	second
SI	meter	kg	second.

Six Basic Units

Quantity	Units	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Current	Ampere	A
Temperature	Degree kelvin	°K
Intensity	candala	Cd.

Derived units

Quantity	Units	Quantity	Units
Volume	m^3	Force	N
Area	m^2	Pressure	N/m^2
Density	kg/m^3	Dynamic Viscosity	$N\text{-}s/m^2$
Discharge	m^3/s	Kinematic Viscosity	m^2/s
Weight Density	N/m^3	Power	Watts
Surface Tension	N/m	Torque	$N\text{-}m$
Momentum	$kg\text{-}m/s$	Work, energy	N-m (or), joule
Velocity	m/s	Thermal conductivity	$W/m\,K$
Angular Velocity	rad/s	Specific heat	$J/kg.K$
Acceleration	m/s^2	Entropy	J/K
Angular Acceleration		Frequency	Hz

Text	Symbol	factor
Tera	T	10^{12}
Giga	G	10^{9}
Mega	M	10^{6}
Kilo	K	10^{3}
Hecto	h	10^{2}
Deci	d	10^{-1}
Centi	c	10^{-2}
Milli	m	10^{-3}
Micro	μ	10^{-6}
Nano	n	10^{-9}
Pico	p	10^{-12}

Physical Properties of Fluid

Density or Mass Density:

- Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted the symbol ρ (rho).
- The unit of mass density in SI unit is kg per cubic meter, i.e., kg/m^3.
- The density of liquids may be considered as constant while that of gases changes with the variation of pressure

Mathematically, mass density is written as

$$\rho = \frac{\text{Mass of fluid}}{\text{Volume of fluid}}$$

Unit — kg/m^3

Dimension — ML^{-3}

Typical Values :— Water — $1000\, kg/m^3$

Air — $1.23\, kg/m^3$ @ standard Pre[ssure] and temperature (

Specific Weight & Weight Density:

- Specific weight & weight density of a fluid is the ratio between the weight of a fluid to its volume.
- Thus weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

Thus mathematically, $w = \dfrac{\text{Weight of fluid}}{\text{Volume of fluid}}$

$$= \frac{\text{Mass of fluid} \times g}{\text{Volume of fluid}}$$

$$w = \rho \times g$$

Unit — N/m^3

Dimension — $ML^{-2}T^{-2}$

Typical values :— Water — $9.81$0 N/m^3

Air — 12

Specific Volume:

Specific Volume of a fluid is defined as the volume of a fluid occupied by a unit mass & Volume per unit mass of a fluid is called Specific volume.

Mathematically, it is expressed as

$$\text{Specific Volume} = \frac{\text{Volume of fluid}}{\text{Mass of fluid}} = \frac{1}{\frac{\text{Mass of fluid}}{\text{Volume}}} = \frac{1}{\rho}.$$

Unit — m^3/kg

Dimenssion — $M^{-1} L^3$

Typical Values :— Water → $10^{-3}\ m^3/kg$

Air → $1.23 \times 10^{-3}\ m^3/kg$.

> Relative Density (Specific Gravity)

- Specific gravity is defined as the ratio of the weight density (& density) of a fluid to the weight density (& density) of a standard fluid.
- For liquids, the standard fluid is taken water and gases, the standard fluid is taken air.
- Specific gravity is also called relative density.
- It is dimensionless quantity and is denoted by the symbol 'S'.

> **Relative density (Specific gravity)**

- Specific gravity is defined as the ratio of the weight density (or density) of a fluid to the weight density of a standard fluid.
- For liquids, the standard fluid is taken water and for gases, the standard fluid is taken air.
- Specific gravity is also called relative density.
- It is dimensionless quantity and is denoted by the symbol 'S'.

Mathematically, S (for liquids) $= \dfrac{\text{Weight density (density) of liquid}}{\text{Weight density (density) of water}}$

Thus weight density of a liquid $= S \times$ Weight density of water

$= S \times 1000 \times 9.81 \ N/m^3$.

The density of a liquid $= S \times$ Density of water

$= S \times 1000 \ kg/m^3$.

Units: Pure number having no units

Dimension: $M^0 L^0 T^0$

Typical values: Mercury – 13.6

Exercise 1

1. Calculate the specific weight, density and specific gravity of one liter of a liquid which weighs 7 N

Given Data:

Volume = 1 litre $= \frac{1}{1000} m^3$ $\left[\because 1 \text{ litre} = \frac{1}{1000} m^3 \right.$

Weight = 7 N

$\left. 1 \text{ litre} = 1000 cm^3 \right]$

Data to be calculated:

Specific Weight (w)

Density (ρ)

Specific gravity (S)

Formula used:

$$\text{Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}}$$

$$\text{Density } (\rho) = \frac{w}{g}$$

$$\text{Specific gravity } (S) = \frac{\text{Density of liquid}}{\text{Density of Water}}$$

$$\text{Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{7N}{\left(\frac{1}{1000}\right) m^3} = 7000 \frac{N}{m^3}.$$

$$= \frac{w}{g} = \frac{7000}{9.81} kg/m^3 = 713.5 kg/m^3$$

$$\text{Specific gravity } (S) = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{713.5}{1000}$$ $\left(\because \text{Density of water} = 10\ldots\right.$

$= 0.7135$.

Result :

Specific Weight(w) = 7000 N/m^3.

Density (ρ) = 713.5 kg/m^3

Specific gravity (S) = 0.7135.

Exercise - 2

10 m^3 of mercury weighs 136×10^4 N. Calculate its Specific weight mass density, specific volume and specific gravity.

Given Data :

Volume = $10 m^3$

weight = 136×10^4

Data to be calculated

Specific weight (w)

Density (ρ)

Specific gravity (S)

Specific volume

Formula used :

$$\text{Specific weight} (w) = \frac{\text{Weight}}{\text{Volume}} \qquad \text{Specific volume} = \frac{1}{\rho}.$$

$$\text{Density} (\rho) = \frac{w}{g}$$

$$\text{Specific gravity} = \frac{\text{Density of liquid}}{\text{Density of Water}}$$

$$\text{Specific weight } (w) = \frac{\text{Weight}}{\text{Volume}} = \frac{136\times10^4}{10} = 136000\ \frac{N}{m^3}.$$

$$\text{Mass density } (\rho) = \frac{w}{g} = \frac{136000}{9.81}\ kg/m^3 = 13863.4\ kg/m^3$$

$$\text{Specific volume} = \frac{1}{\rho} = \frac{1}{13863.4} = 72.13\times10^{-6}\ m^3/kg$$

$$\text{Specific gravity} = \frac{\text{Density of mercury}}{\text{Density of water}} = \frac{13863.4}{1000}$$

$$\boxed{S = 13.86}$$

Result:

Specific weight (w) = $136000\ N/m^3$.

Density (ρ) = $13863.4\ kg/m^3$.

Specific gravity (S) = $72.13\times10^{-6}\ m^3/kg$

Specific volume = 13.86.

Viscosity:

→ Viscocity is defined as the property of a fluid which offers resistance to the movement of one layer of fluid over another adjacent layer of fluid.

→ When two layers of a fluid, a distance apart, move one over the other at different Velocities, say u and u+du as shown in fig the viscosity together with relative velocity cau a shear stress acting between the fluid layers.

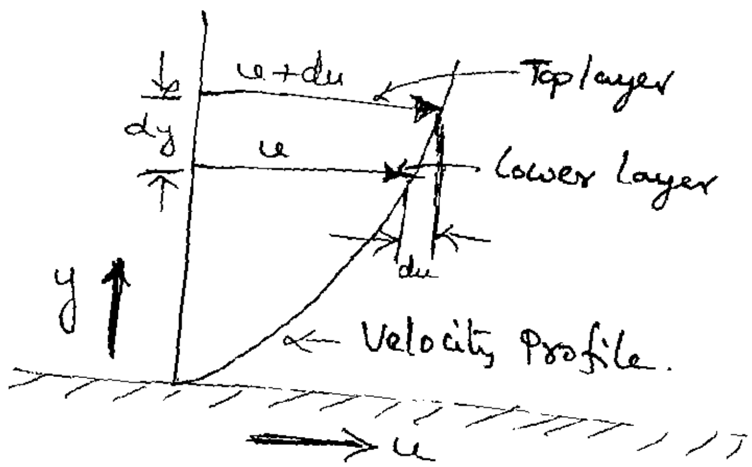


Fig 1: Velocity variation near a solid boundary.

→ The top layer causes a shear stress on the adja-cent lower layer while the lower layer causes a shear stress on the adjacent top layer.

→ This shear stress is proportional to the rate of change of velocity with respect to y.

→ It is denoted by the symbol called Tau.

Mathematically,

$$\tau \propto \frac{du}{dy} \quad \text{or} \quad \tau = \mu \cdot \frac{du}{dy}.$$

where μ (called mue) is the constant of proportionality and is known as the co-efficient of dynamic viscosity or only viscosity. $\frac{du}{dy}$ Represents the rate of shear strain or rate of shear deformation or velocity gradient.

$$\text{we have } \mu = \frac{\tau}{\left(\frac{du}{dy}\right)}$$

Thus viscocity is also defined as the shear stress required to produce unit rate of shear strain

units of viscosity.

→ The units of viscosity is obtained by putting the dimensions of the quantities.

$$\mu = \frac{\text{Shear stress}}{\frac{\text{Change of Velocity}}{\text{change of distance}}} = \frac{\text{Force/Area}}{\left(\frac{\text{length}}{\text{Time}}\right) \times \frac{1}{\text{length}}}$$

$$= \frac{\text{Force}/(\text{length})^2}{\frac{1}{\text{Time}}} = \frac{\text{Force} \times \text{Time}}{(\text{length})^2}.$$

→ In MKS system, force is represented by kgf and length by meter (m), in CGS system, force is represented by dyne and length by cm and in SI system force is represented by Newton (N) and length by meter (m).

$$\therefore \text{ MKS unit of Viscosity} = \frac{\text{kgf-sec}}{m^2}.$$

$$\text{CGS unit of Viscosity} = \frac{\text{dyne-sec}}{cm^2}.$$

In the above expression N/m^2 is also known as Pascal which is represented by Pa. Hence $\frac{N}{m^2}$ = Pa = Pascal.

$$\therefore \text{ SI unit of Viscosity} = \frac{Ns}{m^2} = \underline{\underline{Pa.S}}$$

$$\text{SI unit of Viscosity} = \frac{\text{Newton-sec}}{m^2} = \frac{Ns}{m^2}.$$

The unit of viscocity in CGS is also called Poise. which is equal to $\frac{\text{dyne-sec}}{cm^2}$.

The Numerical conversion of the unit of viscocity from MKS unit to CGS unit is given below:

$$\text{one } \frac{\text{kgf-sec}}{m^2} = \frac{9.81 \text{N-sec}}{m^2} \quad \left[\because 1 \text{kgf} = 9.81 \text{ Newton}\right]$$

But one Newton = one kg (mass) × one $\left(\frac{m}{sec^2}\right)$ accelerati

$$= \frac{(1000 \text{ gm}) \times (100 \text{ cm})}{sec^2} = 1000 \times 100 \frac{\text{gm-cm}}{sec^2}$$

$\boxed{1N = 1000 \times 100 \text{ dyne}}$

$\therefore$ one $\frac{kgf\text{-}Sec}{m^2} = 9.81 \times 100000 \ \frac{dyne\text{-}sec}{m^2}$

$$= 9.81 \times 100000 \ \frac{dyne\text{-}Sec}{100 \times 100 \times cm^2}$$

$$= 9.81 \times 10 \frac{dyne - sec}{cm^2}$$

one $\frac{kgf\text{-}Sec}{m^2} = \frac{98.1 \ dyne\text{-}Sec}{cm^2} = \underline{\underline{98.1}}$ poise

one $\frac{Ns}{m^2} = \frac{98.1}{9.81}$ poise $= 10$ poise & one Poise $= \frac{1}{10} \frac{Ns}{m^2}$

Note:

(i) In SI units second is represented by 's' and not by sec

(ii) If viscosity is given in poise, it must be divided by 10 to get its equivalent numerical value in SI units.

Sometimes a unit of viscosity as centipoise is used

where 1 Centipoise $= \frac{1}{100}$ Poise (or) $1 \ cP = \frac{1}{100} P$.

(cP = centipoise, P = Poise)

The viscosity of water at 20°C is 0.01 Poise

(or)

1.0 centipoise.

Kinematic Viscosity:

→ It is defined as the ratio between the dynamic viscosity and density of fluid.

→ It is denoted by the Greek symbol (ν) called 'nu'. Thus mathematically

$$\nu = \frac{\text{Viscosity}}{\text{Density}} = \frac{\mu}{\rho} \rightarrow (3)$$

The units of kinematic viscosity is obtained as

$$\nu = \frac{\text{units of } \mu}{\text{units of } \rho} = \frac{\text{Force} \times \text{Time}}{(\text{Length})^2 \times \frac{\text{Mass}}{(\text{Length})^3}} = \frac{\text{Force} \times \text{Time}}{\frac{\text{Mass}}{\text{Length}}}$$

$$\nu = \frac{\text{Mass} \times \frac{\text{Length}}{(\text{Time})^2} \times \text{Time}}{\left(\frac{\text{Mass}}{\text{Length}}\right)} \qquad \left\{ \because \text{Force} = \text{Mass} \times \text{Accelaration} = \text{Mass} \times \frac{\text{Length}}{\text{Time}^2} \right\}$$

$$\nu = \frac{(\text{Length})^2}{\text{Time}}$$

→ In MKS and SI, the unit of kinematic viscosity is metre²/sec & m²/sec while in CGS units it is written as cm²/s

→ In CGS units, kinematic viscosity is also known stoke.

Thus, one stoke $= cm^2/s = \left(\frac{1}{100}\right)^2 m^2/s = 10^{-4}\ m^2/s$

Centistoke means $= \frac{1}{100}$ stoke.

Newton's Law of Viscosity:

- It states that the shear stress on a fluid element layer is directly proportional to the rate of shear strain.
- The constant of proportionality is called the coefficient of Viscos
- Mathematically, it is expressed as given by equation (1) & as

$$\tau = \mu \frac{du}{dy}$$

- Fluids which obey the above relation are known as Newtonian Fluids and the Fluids which do not obey the above relation are called Non - Newtonian Fluids.

Variation of Viscosity with Temperature

- Temperature affects the Viscosity.
- The viscosity of liquids decreases with the increase of temperature while the viscosity of gases increases with the increase of temperature
- This is due to reason that the viscous forces in a fluid are due to cohesive forces and molecular momentum transfer.
- In liquids the cohesive forces predominates the molecular momentum transfer, due to closely packed molecules & with the increase in temperature, the cohesive forces decreases with the result of decreasing viscosity.
- But in case of gases the cohesive force are small and molecular momentum transfer predominates.
- With the increase in temperature, molecular momentum transfer increases and hence viscosity increases.
- The relation between viscosity and temperature for liquids and gases are:

(i) For Liquids $\mu = \mu_0 \left(\frac{1}{1 + \alpha t + \beta t^2} \right)$ → (i) where μ = Viscosity of liquid at $t^\circ C$, in poise

μ_0 = Viscosity of liquid at $0^\circ C$, in poise

α, β = are constants for the liquid.

For water, $\mu_0 = 1.79 \times 10^{-3}$ poise

$\alpha = 0.03368$ & $\beta = 0.000221$.

The equation (i) shows that with the increase of temperat the viscosity decreases.

(ii) For a gas, $\mu = \mu_0 + \alpha t - \beta t^2 \longrightarrow$ (ii)

where for air $\mu_0 = 0.000017$

$\alpha = 0.000000056$

$\beta = 0.1189 \times 10^{-9}$

The equation (ii) shows that with the increase of temperature, the viscosity increases.

Classification of Fluids:

The fluids may be classified into the following five types :

1) Ideal Fluid
2) Real Fluid
3) Newtonian Fluid
4) Non-Newtonian Fluid
5) Ideal plastic Fluid

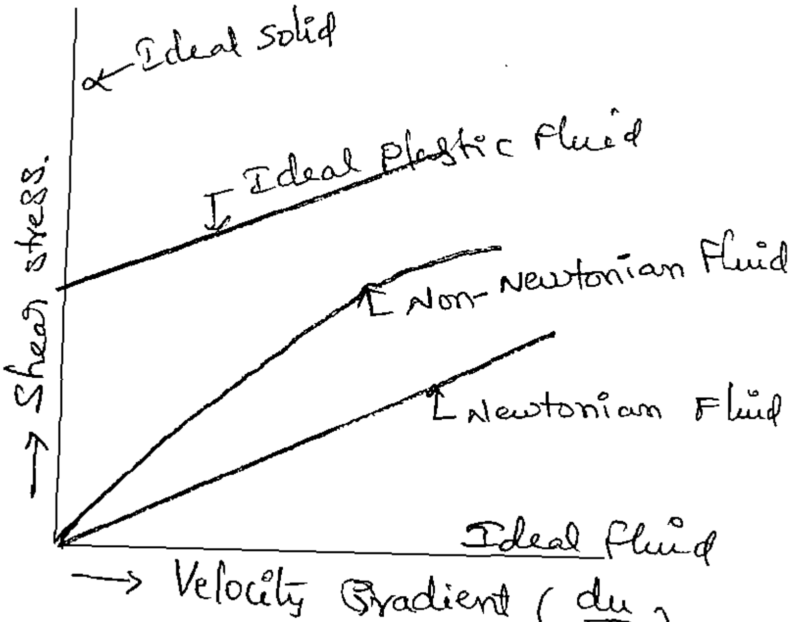


Fig: Types of Fluids

Ideal Fluid: A Fluid, which is incompressible and is having no viscosity, is known as an ideal fluid. Ideal fluid is only an imaginary fluid as all the fluids, which exist, have some viscosity.

Real Fluid: A fluid, which possesses Viscosity, is known as real fluid. All the fluids, in actual practice, are real fluid

Newtonian Fluid: A real fluid, in which the shear stress is directly, proportional to the rate of shear strain (or Velocity gradient), is known as a Newtonian Fluid.

Non-Newtonian Fluid: A real fluid, in which the shear stress is not propotional to the rate of shear strain (or Velocity gradient). known as a Non-Newtonian Fluid.

Ideal Plastic Fluid: A Fluid, in which shear stress is more than the yield value of shear stress is proportional to the rate of shear strain is known as Ideal Plastic Fluid

Problems

1. The space between two square parallel plates is filled with oil. Each side of the plate is 60mm. The thickness of the oil film is 12.5mm. The upper plate, which moves at 2.5 meter per sec requires a force of 98.1 N to maintain the speed. Determin

i) The dynamic viscosity of the oil in poise, and

ii) The kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

Given Data:

- Each side of a square plate = 60 mm = 0.60m
- Thickness of oil film, $dy = 12.5\,mm = 12.5 \times 10^{-3} m$.
- Velocity of upper plate, $u = 2.5\,m/sec$
- Force required on upper plate, $F = 98.1\,N$

Data to be calculated:

i) The dynamic viscosity of the oil in poise, and

ii) The kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.

Formula used:

Shear stress $\tau = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A}$

Dynamic viscosity of oil $\tau = \mu \dfrac{du}{dy}$

Solution:

Each side of a square plate = 60cm = 0.60m

∴ Area, $A = 0.6 \times 0.6 = 0.36\,m^2$

Thickness of oil $dy = 12.5\,mm = 12.5 \times 10^{-3}$

Velocity of upper plate, $u = 2.5\, m/sec$

$\therefore$ Change of velocity between Plates, $du = 2.5\, m/sec$

Force required on upper plate, $F = 98.1\, N$

$\therefore$ Shear stress, $\tau = \frac{Force}{Area} = \frac{F}{A} = \frac{98.1\, N}{0.36\, m^2}$

i) Let μ = Dynamic Viscosity of oil

$$\tau = \mu \cdot \frac{du}{dy} \quad \& \quad \frac{98.1}{0.36} = \mu \times \frac{2.5}{12.5 \times 10^{-3}}$$

$$\mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635\, \frac{NS}{m^2} \quad \left(\because 1\, \frac{NS}{m^2} = 10\, poise\right)$$

$$= 1.3635 \times 10 = 13.635\ poise.$$

ii) Sp. gr. of oil, $S = 0.95$

Let ν = kinematic Viscosity of oil.

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950\, kg/m^3$

Using the relation, $\nu = \frac{\mu}{\rho}$, we get $\nu = \frac{1.3635\left(\frac{NS}{m^2}\right)}{950}$

$$\nu = 0.001435\, m^2/sec = 0.001435 \times 10^4\, cm^2/sec$$

$$= \underline{14.35}\ Stokes \quad \left(\because cm^2/sec = Stoke\right)$$

Result:

- Dynamic Viscosity of oil = 13.635 poise.
- The kinematic Viscosity of the oil in stokes if the specific gravity of the oil is 0.95 = 14.35 stokes.

1. If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which 'u' is the velocity in meter per second at a distance y meter above the plate, determine the shear stress at $y=0$ and $y=0.15$ m. Take dynamic viscosity of fluid as 8.63 poises.

Given data:

Velocity distribution $u = \frac{2}{3}y - y^2$

$$\therefore \frac{du}{dy} = \frac{2}{3} - 2y$$

Dynamic Viscosity $= 8.63$

Data to be calculated

Shear stress at $y=0$ and $y = 0.15$m

Formula used:

Shear stress at $y=0$ is given by $\tau_0 = \mu \left(\frac{du}{dy}\right)_{y=0}$

Shear stress at $y = 0.15$ m is given by

$$(\tau)_{y=0.15} = \mu \left(\frac{du}{dy}\right)_{y=0.15}$$

Solution:

$$u = \frac{2}{3}y - y^2$$

$$\therefore \frac{du}{dy} = \frac{2}{3} - 2y$$

$\left(\frac{du}{dy}\right)_{at\ y=0}$ & $\left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} - 2(0) = \frac{2}{3} = 0.667$

Also $\left(\frac{du}{dy}\right)_{at\ y=0.15}$ & $\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2(0.15)$

$= 0.367$

Value of $\mu = 8.63$ poise $= \frac{8.63}{10}$ SI units $= 0.863 \frac{N.s}{m^2}$

Now shear stress is given by equation as

$$\tau = \mu . \frac{du}{dy}$$

i) shear stress at $y = 0$ is given by

$$\tau_0 = \mu\left(\frac{du}{dy}\right)_{y=0} = 0.863 \times 0.667 = 0.5756 \frac{N}{m^2}$$

ii) Shear stress at $y = 0.15m$ is given by

$$(\tau)_{y=0.15} = \mu\left(\frac{du}{dy}\right)_{y=0.15} = 0.863 \times 0.367 = 0.3167 \frac{N}{m^2}$$

Result :

Shear stress at $y = 0$ $= 0.5756 \frac{N}{m^2}$

Shear stress at $y = 0.15 = 0.3167 \frac{N}{m^2}$

Cohesion:-

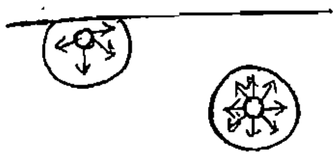
It is defined as the intermolecular attraction b/n the same kind of molecules.

Adhesion:-

It is defined as the intermolecular attraction b/n the different kind of molecules is known as Adhesion.

6, **Surface Tension:-**

It is defined as the property of fluid by virtue of which the surface of liquid takes small loads like paper, card and light weight materials.



(or)

It is defined as the force acting on the surface of liquid per unit length.

Surface Tension is mainly due to cohesion and intermolecular attraction b/n the liquid particles. It is inversely propotional to temperature. It is denoted by "σ".

$$\sigma = \frac{\text{Force}}{\text{unit length}} = \frac{MLT^{-2}}{L} = ML^{0}T^{-2}$$

Some practical Examples:-

- The ~~spherical~~ Spherical shape of rain water droplet is due to surface tension.
- The spherical shape of soap bubble is also due to surface tension.
- The property that is utilized in the manufacture of led shots used in guns is due to surface tension.
- Ants and small insects will walk on the surface of liquid because of surface tension.

* The surface tension of water at $20^{\circ}c$ is 0.074 N/m.

* The surface tension of Mercury at $20^{\circ}c$ is 0.48 N/m

→ The pressure inside the soap bubble is given by the formulae $p = \frac{8\sigma}{D}$ (or) $\frac{4\sigma}{r}$

where, σ is known as surface Tension

→ The pressure inside the water bubble is given by the formula $P = \frac{4\sigma}{D}$ (or) $\frac{2\sigma}{r}$.

→ The pressure inside the water jet is given by the formula $P = \frac{2\sigma}{D}$ (or) $\frac{\sigma}{r}$.

7, Capillarity:-

capillarity is defined as the property by virtue of which the liquid raises & falls in a thin glass tube when dipped in the vessel containing that liquid. The property of capillarity is due to both cohesion and Adhesion.

If Adhesion is more than cohesion capillary ~~ra~~ rise occurs with concavity downwards.

If cohesion is more than Adhesion capillary fall occurs with concavity upwards.

In case of water capillary rise occurs. The formula to calculate the capillary rise/fall is given by $h = \frac{4\sigma \cos\theta}{\omega \cdot d}$

where, h = capillary rise or fall.

ω = Specific wt. of liquid

θ = Contact angle b/n liquid & glass.

d = diameter of the tube.

σ = Surface Tension of water in contact with air

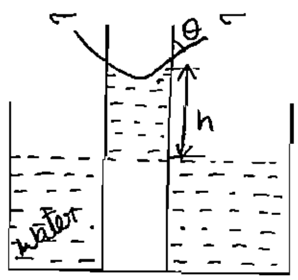
Contact angle b/n water and glass is 0°

contact angle b/n glass and mercury is 130°.

The surface of soil even above the water table is wet due to the capillary action of soil force.

The units for capillarity is mm/cm/m.

* Expression for capillary rise :-



consider a glass tube of small diameter 'd' open at both ends and is inserted in a liquid say water. The liquid will rise in the tube above the level of the liquid.

Let h = capillary rise i.e., height of the liquid in the tube above the surface of the liquid.

For equilibrium condition the weight of the liquid in the capillary rise should be balanced by surface Tension force.

σ = Surface tension.

→ vertical component of surface tensile force

$$= \sigma \times \text{circumference} \times \cos\theta$$

$$= \sigma \times \pi d \times \cos\theta \rightarrow ① \uparrow \text{(upward force)}$$

→ weight of liquid in capillary rise

= volume × Specific weight

= volume of the liquid × w

$$= \frac{\pi d^2}{4} \times h \times w \qquad (w = \rho g)$$

$$w = \rho . g \times \frac{\pi d^2}{4} \times h \rightarrow ② \downarrow \text{(Downward force)}$$

For equilibrium condition upward force is equal to the downward force. i.e., ① = ②

$$\sigma \times \cancel{\pi}\cancel{d} \times \cos\theta = \rho . g \times \frac{\cancel{\pi} d^{\cancel{2}}}{4} \times h$$

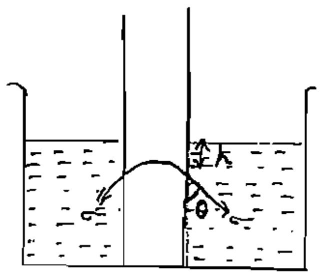
$$\therefore h = \frac{4\sigma\cos\theta}{\rho g d}$$

For water $\theta = 0$

$\therefore \cos\theta = 1$

$$\therefore h = \frac{4\sigma}{\rho g d}$$

* Expression for capillary fall :-



consider a glass tube of small diameter 'd' open at both the ends is dipped ~~in~~ into a liquid say mercury. The liquid will fall in the tube below the level of the mercury.

Two forces are acting on the mercury inside the tube.

1, Due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos\theta \downarrow$ — (1)

2, Due to hydrostatic force acting in upward direction and is equal to intensity of pressure at a depth $= h$

$$p = \omega h$$

$$p = \rho g h$$

Hydrostatic force = $p \times$ Area

$$= \rho g h \times \frac{\pi d^2}{4} \quad — ②$$

$$\sigma \times \pi d \times \cos\theta = \rho g h \frac{\pi d^2}{4}$$

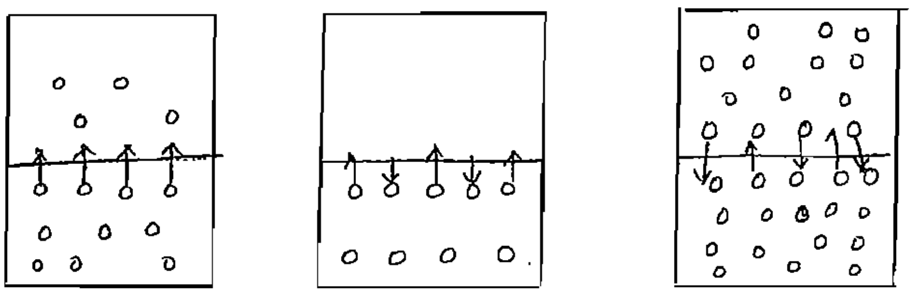
$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

For mercury $\theta = 130°$

$$\therefore \cos\theta = -0.6428$$

$$\therefore h = \frac{-4\sigma (0.6428)}{\rho g d}$$ (– indicates fall)

8, Vapour pressure :-



If a liquid is placed inside a container and confined at all sides. Let a constant temperature be maintained within the container. Some of the liquid molecules have sufficient energy to break away from the liquid surface and enter the air space in vapour state. After certain time the air will contain enough liquid molecules to exert a partial pressure of the air on the surface of the liquid. This pressure forces the vapour molecules to rejoin the liquid surface. This pressure is known as vapour pressure.

Saturated vapour pressure:-

At equilibrium condition i.e., the rate ~~of~~ at which the molecules re-enter the liquid is equal to the rate at which they leave the surface.

Impt At equilibrium condition the pressure exerted by the vapour on the surface of the liquid is known as saturation vapour pressure

Points to be remembered:-

- As the molecular activity increases with the temperature, the vapour pressure also increases with rise in temperature.
- Boiling of the liquid starts when the ~~ete~~ external pressure is imposed on the liquid, is equal to (or) less than vapour pressure

- The saturation vapour pressure is of great practical importance in solving the fluid problems. If the pressure at any point in the fluid approaches the vapour pressure the liquid starts boiling vapour bubbles are form in the region of low pressure.
- In order ~~of~~ to avoid cavitation pressure is not permitted to fall below 0.25 kg/cm^2 absolute.
- The basic property of the fluid i.e., taken into consideration to decide whether a liquid can be used as manometric liquid or not is vapour pressure.
- Mercury passes very low pressure and is equal to 0.173 N/m^2 vapour pressure of water is 2447 N/m^2
- volatile substances like Benzene and petrol are assumed to passes very high vapour pressure.

9, Compressibility:-

The compressibility is the measure of change of volume / density when a substance is subjected to pressure. A fluid may be compressed by the application of pressure. such a compressed fluid will expand to it original volume when the applied pressure is withdraw. But for all practical purposes water is considered to be practically incompressable.

Mathematically can be expressed as

$$\text{compressability} = \frac{\text{change in volume}}{\text{change in pressure}} = \frac{\Delta V}{\Delta P}$$

sometimes compressibility may be expressed in terms of bulk modulus.

$$\text{Bulk modulus} = \frac{\text{pressure}}{\text{volumetric strain}} = \frac{P}{\Delta V/V}$$

It is denoted by "K" units are N/mm^2.

Bulk modulus of ~~elea~~ elasticity is directly proportional to pressure and inversely proportional to temperature

- Bulk modulus of elasticity of water is $2.1 \times 10^{-3}\ N/mm^2$ (or) $2.1 \times 10^{-2}\ kg/cm^2$ ($\because$ 1 cm = 10 mm & 1 kg = 10 N)
- Air is 20,000 times ~~more~~ more compressable than water.

① Calculate the Specific weight, density & Specific gravity of 1 litre of liquid which weighs 7N.

Sol:- $W = \frac{7N}{\frac{1}{1000}}$

$1m^3 = 1000 lit$

$1 lit = 1/1000 \, m^3$

$W = 7N$

$= 7000 \, N/m^3$.

$w = \rho \times g$

$\rho = \frac{w}{g} = \frac{7000}{9.81} = 713.5 \, Kg/m^3$.

$S = \frac{713.5}{1000} = 0.7135 \, N/m^3$ (or) $\frac{7000}{9810} = 0.7135 \, N/m^3$.

② Calculate the density, specific weight and weight of 1 litre of petrol.

Sol:- Specific gravity of petrol = 0.7

$G = \frac{\rho_p}{\rho_w}$

$\rho_p = G \cdot \rho_w \Rightarrow 0.7 \times 1000 = 700 \, Kg/m^3$.

$w = \rho \times g$

$w = 700 \times 9.81 = 6867 \, N/m^3$.

$w = \frac{W_p}{V}$

$W_p = V \times w$

$= 6867 \times 1/1000$

$= 6.867 \, N$

$= 0.686 \, Kg$

(3) Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.

Sol:- $\nu = \frac{\mu}{\rho}$

$\nu = 6\,\text{stokes} = 6\,cm^2/sec = 6 \times 10^{-4}\,m^2/sec.$

$G = \frac{\rho_l}{\rho_w}$

$\rho_l = 1.9 \times 1000 = 1900\,Kg/m^3.$

$6 \times 10^{-4} = \frac{\mu}{1900}$

$\therefore \mu = 1.14\,\frac{N\text{-}S}{m^2}$

$\therefore \mu = 11.4$ poise.

Note:- $1\,\text{poise} = 1/10 \times \frac{N\text{-}S}{m^2}$

$1N = Kg\text{-}m/sec^2$

$\frac{Kg}{m^3} \times \frac{m^2}{sec}$

$\frac{Kg}{m \cdot sec} \times \frac{sec}{sec} \times \frac{m}{m}$

$\frac{Kg\text{-}m}{sec} \cdot \frac{sec}{m^2}$

$\frac{N-S}{m^2} \Rightarrow \frac{N\text{-}S}{m^2} \times 10 = \text{poise}.$

(4) Determine the bulk modulus of elasticity of liquid if the pressure of the liquid increased from 70 N/cm² to 130 N/cm². The volume of the liquid decreases by 0.15%.

Sol:- $I.p = 70\,N/cm^2$

$F.p = 130\,N/cm^2$

$\Delta p = 130 - 70 \Rightarrow 60\,N/cm^2.$

$\Delta V = \frac{0.15}{100}$

$B.M = K = \frac{\Delta p}{\frac{0.15}{100}} = \frac{60}{0.15} \times 100 = \frac{\overset{4}{\cancel{6}}\cancel{0}0000}{\cancel{15}}$

$\Rightarrow 40000\,N/cm^2.$

(5) What is bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of $0.0125\,m^3$ at wt. $80\,N/cm^2$ in a volume of $0.0124\,m^3$ at $150\,N/cm^2$ pressure.

Sol:- $$B \cdot M = \frac{\Delta p}{\frac{\Delta V}{V}} = \frac{150-80}{\frac{0.0125-0.0124}{0.0125}}$$

$$= 8750\,N/cm^2.$$

(6) Calculate the capillary effect in mm in the glass tube of 4mm diameter when immersed in 1. water 2. mercury. The temperature of liquid is $20^\circ c$ and the values of S.T of water & mercury at $20^\circ c$ in contact with air are $0.0736\,N/m$ and $0.51\,N/m$ respectively. The angle of contact of water is 0° and that of mercury is 130°.

Sol:- $$h = \frac{4\sigma\cos\theta}{\rho \cdot g \cdot d}$$

(1) $$h = \frac{4 \times 0.0736 \times \cos 0}{1000 \times 9.81 \times 4 \times 10^{-3}} = 7.51 \times 10^{-3}\,m = 7.51\,mm$$

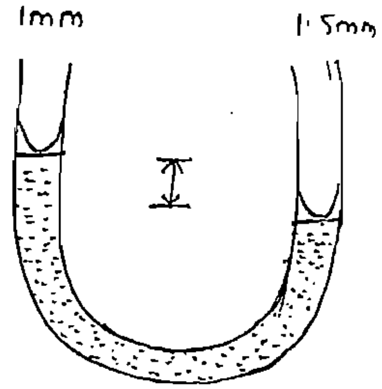
$$13.6 = \frac{\rho_m}{\rho_w}$$

$$\rho_m = 13.6 \times 1000$$

(2) $$h = \frac{4 \times 0.51 \times \cos 130}{13.6 \times 9.81 \times 4 \times 10^{-3}}$$

$$= -2.45 \times 10^{-3}\,m \Rightarrow -2.45\,mm$$ (- indicates fall)

(7) A U tube is made of two capillaries of diameters 1mm and 1.5mm respectively. The tube is kept vertically and partially filled with water. The S.T of the water is 0.0736 N/m. Calculate the difference in the levels of minuscus caused by the capillarity.



Sol:-

$$h_1 = \frac{4 \times 0.0736 \times 1}{1000 \times 9.81 \times 1 \times 10^{-3}}$$

$= 0.03 \Rightarrow 30mm$

$$h_2 = \frac{4 \times 0.0736 \times 1}{1000 \times 9.81 \times 1.5 \times 10^{-3}}$$

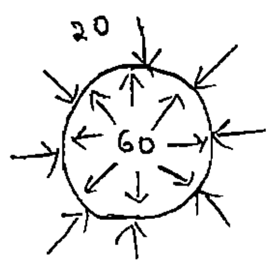
$= 20mm$

$\therefore h_1 - h_2 = 30mm - 20mm = 10mm.$

(8) The S.T of water in contact with air at 20°c is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure calculate the diameter of the droplet of water.

Sol:- p = pressure intensity in excess of out-side pressure.

$\Rightarrow$ Inside pressure − out-side pressure.



water bubble.

$$p = \frac{4\sigma}{d}$$

$p = 0.02\, N/cm^2 = 0.02 \times 10^{-4}\, N/m^2$

$\sigma = 0.0725\, N/m$

$$0.02 \times 10^{-4} \Rightarrow \frac{4 \times 0.0725}{d}$$

$d = 1.45mm$

$= 1.45mm.$

$0.02\, N/cm^2$
$= 0.02 \times 10^{-4}\, N/m^2$

(9) Find the S.T in a soap bubble of 40mm diameter when the inside pressure is $2.5 \, N/m^2$.

Sol:-

$$p = \frac{8\sigma}{d}$$

$$2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\therefore \sigma = 0.0125 \, N/m.$$

(10) The pressure outside the droplet of water of diameter 0.04mm is $10.32 \, N/m^2$ calculate the pressure within the droplet If S.T is given by $0.0725 \, N/m$.

Sol:- water bubble:-

$$P = \frac{4\sigma}{d}$$

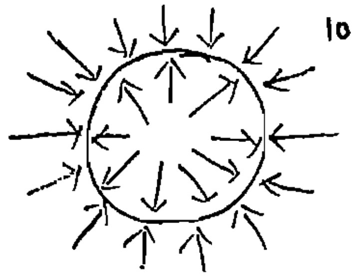
$$P = \frac{4 \times 0.0725}{0.04 \times 10^{-3}}$$

$$P = 7250 \, N/m^2$$

$$P = \frac{7250}{10^4} = 0.725 \, N/cm^2.$$

$$\therefore I.P = 10.32 + 0.725$$

$$= 11.045 \, N/cm^2.$$



$$p = I.p - O.p$$

$$I.p = p + O.p.$$

(1) Find the kinematic viscosity of oil having density $981\ kg/m^3$. The shear stress at a point in oil is $0.2452\ N/m^2$ and velocity gradient at that point is 0.2 per second.

Sol:-

$$\nu = \frac{\mu}{\rho}$$

$$\tau = \mu \cdot \frac{du}{dy}$$

$$0.2452 = \mu \cdot 0.2$$

$$\therefore \mu = 1.226\ \frac{N\text{-}s}{m^2}$$

$$\nu = \frac{1.226}{981}$$

$$= 1.249 \times 10^{-3}$$

$$= 0.125 \times 10^{-2}\ m^2/sec$$

$$= 0.125 \times 10^{-2} \times 10^{4}\ cm^2/sec$$

$$= 0.125 \times 10^{2}\ cm^2/sec$$

$$= 0.125 \times 10^{2}\ \text{stokes}.$$

$$\nu = 12.5\ \text{stokes}.$$

(2) Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes

(A)

$$\mu = 0.05\ \text{poise} = \frac{0.05}{10}\ N\text{-}s/m^2.$$

$$= 0.005\ N\text{-}s/m^2.$$

$$\nu = 0.035\ \text{stokes}$$

$$= 0.035\ cm^2/sec$$

$$= 0.035 \times 10^{-4}\ m^2/sec.$$

$$G = \frac{\text{density of fluid}}{\text{density of water}}.$$

$$\nu = \frac{\mu}{\rho}$$

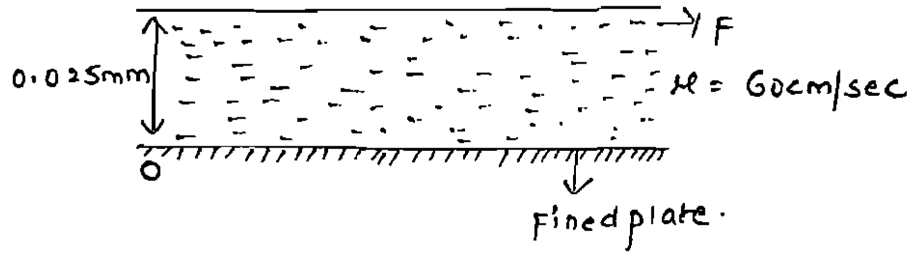
$$\rho = \frac{\mu}{\nu} = \frac{0.05}{10 \times 0.035 \times 10^{-4}} = 1428.5\ kg/m^3.$$

$$\therefore G = \frac{1428.5}{1000} = 1.428$$

$$\Rightarrow 1.43. \quad \therefore \boxed{G = 1.43}$$

③ A plate 0.025mm distance from a fixed plate moves at 60cm/sec and he finds a force of 2N per unit area ($2N/m^2$) to maintain the speed. Determine the fluid viscosity between the plates.

Sol:-



$$\tau = \mu \cdot \frac{du}{dy}.$$

$$2 = \mu \times \frac{0.6}{0.025 \times 10^{-3}}.$$

$$\therefore \mu = 8.33 \times 10^{-5}\ N\text{-}S/m^2$$

$$\mu = 8.33 \times 10^{-5} \times 10 \text{ poise.}$$

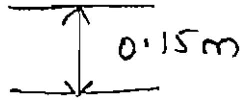
$$\therefore \mu = 8.33 \times 10^{-4} \text{ poise.}$$

④ A flat plate of Area $1.5 \times 10^6\ mm^2$ is pulled with a speed of 0.4m/s relative to another plate located at a distance of 0.15mm from it. Find the force and power required to maintain a speed. If the fluid separating them is having viscosity as 1 poise.

Sol:-

$A = 1.5 \times 10^{+6}\ mm^2$

$A = 1.5\ m^2$



$du = 0.4 m/s.$

$dy = 0.15mm = 0.15 \times 10^{-3} m$

$\mu = 1$ poise

$\therefore \mu = 1/10\ \frac{N\text{-}S}{m^2} = 0.1\ \frac{N\text{-}S}{m^2}.$

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\tau = 0.1 \times \frac{0.4}{0.15 \times 10^{-3}}$$

$$\therefore \tau = 266.66\ N/m^2.$$

$\therefore$ Force $= \tau \times$ Area

$= 266.66 \times 1.5$

$= 400 N.$

$\therefore$ Power = Force $\times$ Velocity

$= 400 \times 0.4$

$= 160$ watt.

(5) Determine the intensity of shear of an oil having viscosity 1 poise. The oil is used for lubricating the clearance between the shaft of dia 10cm and its general bearing. The clearance is 1.5mm and the shaft rotates at 150 rpm.

Sol:- $\mu = 1$ poise $= 1/10\ N\cdot s/m^2.$

$D = 10cm = 0.1m$

$dy = 1.5\ mm = 1.5 \times 10^{-3} m$

$N = 150 rpm.$

$\tau = \mu \cdot \frac{du}{dy}.$

Tangential velocity $= du = u = \frac{\pi D N}{60}$

$= \frac{\pi \times 0.1 \times 150}{60} = 0.785\ m/sec.$

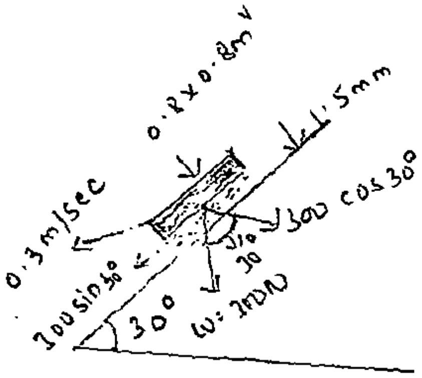
Shear stress $(\tau) = 1/10 \times \frac{0.785}{1.5 \times 10^{-3}}$

$= 52.33\ N/m^2.$

(6) Calculate the dynamic viscosity of an oil which is used for lubrication between a square plate of size 0.8m x 0.8m and an inclined plane with an angle of inclination 30 as shown in figure. The wt of the square plate is 300N and it slides down the inclined plate with uniform velocity of 0.3 m/s. The thickness of oil film is 1.5mm.

Sol:- $\tau = \mu \cdot \frac{du}{dy}$

$\tau = \frac{S.F}{Area}$ (Shear force).



$= \frac{300 \times \sin 30^\circ}{0.8 \times 0.8}$

$= 234.37 \ N/m^2$.

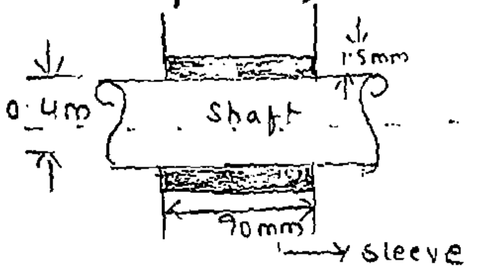
$\therefore 234.37 = \mu \cdot \frac{0.3}{1.5 \times 10^{-3}}$.

$\therefore \mu = 1.17 \ N\text{-}s/m^2$

$\mu = 1.17$ poise.

(7) The Dynamic viscosity of an oil used for lubrication between a shaft and sleeve is 6 poise. The shaft is of dia 0.4m and rotates at 190 rpm. calculate the power loss in the bearing for a sleeve length of 90mm. The thickness of oil film is 1.5mm.

Sol:-



N = 190 rpm

μ = 6 poise.

Power lost $= \Upsilon \times \omega$ (Torque x angular velocity).

$= \Upsilon \times \frac{2\pi N}{60}$

Torque $= \Upsilon =$ shear force $\times \frac{D}{2}$.

Shear force $=$ shear stress $\times$ circumferential Area $\times D \times L$

$$\tau = \mu \cdot \frac{du}{dt}.$$

$$du = V = \frac{\pi DN}{60}$$

$$= 1592 \times \pi \times 0.4 \times 90 \times 10^{-3}$$

$$= 180.05 N.$$

$\therefore$ power lost $= \frac{2\pi \times 190 \times 36.01}{60}$

$= 716.12$ watt

$$du = \frac{\pi \times 0.4 \times 190}{60}$$

$= 3.98$ m/sec.

$$\tau = 6/10 \times \frac{3.98}{1.5 \times 10^{-3}}$$

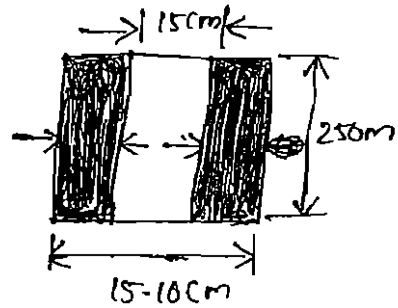
$= 1592$ N/m^2.

$$T = 180.05 \times \frac{0.4}{2}$$

$= 36.01$ N-m.

2) A 15cm diameter vertical cylinder rotates concentratically inside the another cylinder of dia 15.10cm. Both cylinder are 25cm high. The space b/t the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12 N/m is required to rotate the inner cylinder at 100rpm det the viscosity of the fluid.

Sol



$$u = \frac{\pi D N}{60}$$

$$= \frac{\pi \times 0.15 \times 100}{60}$$

$$= 0.78 \text{ m/sec}$$

$$T = S.F \times \frac{D}{2}$$

$$12 = S.F \times \frac{0.15}{2}$$

$$S.F = 160 N$$

$$\tau = \frac{160}{0.15 \times \pi \times 0.25}$$

$$\tau = 1358.12 \text{ N/m}^2$$

$$\tau = \mu . \frac{du}{dy}$$

$$1358 = \mu . \frac{0.78}{0.0005}$$

$$\mu = 0.865 \text{ N-s/m}^2$$

$$\therefore \mu = 8.65 \text{ poise}$$

$$dy = \frac{15.10 - 15.0}{2}$$

$$= \frac{0.10}{2}$$

$$= 0.05 \text{cm}$$

$$= 0.0005 m$$

3) If the velocity distribution over a plate is given by $u = \frac{2}{3}y - y^2$ in which u is the velocity in m/sec at a distance ~~y~~ 4m above the plate det the shear stress at $y=0$ and $y = 0.115$ m. Take Dynamic viscosity of fluid as 8.63 poise

$$y = 0$$

$$y = 0.15$$

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{2}{3} - 2y$$

$$\left(\frac{du}{dy}\right)_{y=0} = \frac{2}{3} = 0.667$$

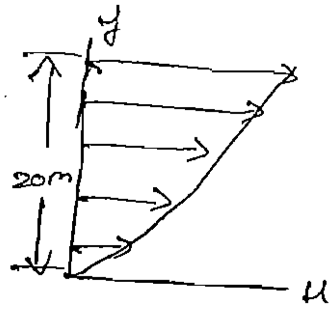
$$\left(\frac{du}{dy}\right)_{y=0.15} = \frac{2}{3} - 2 \times 0.15 = 0.366$$

$$y = 0 \quad \tau = \frac{8.6}{10} \times 0.667 = 0.57 \, N/m^2$$

$$y = 0.15 \quad \tau = \frac{8.6}{10} \times 0.366 = 0.316 \, N/m^2$$

4) If the velocity profile of a fluid over a plate is parabolic with the vertex 20cm from the plate where the velocity is 120 m/sec cal, the velocity gradiants & shear stress at a distances of 0.10 & 20cm from plate. If viscosity of a fluid is 8.5 poise

Sol: Boundary Conditions:



1) at $y=0$ $u=0$

2) at $y=20cm$ $u=120$ cm/sec

3) at $y=20cm$ $\frac{du}{dy}=0$

$$u = ay^2 + by + c \rightarrow ①$$

substituting the boundary condition 1 in ① we get

$$0=c$$

substituting boundary condition 2 in ① we get

$$120 = a\times(120)^2 + b(20) + 0$$

$$120 = 400a + 20b$$

applying 3rd boundary condition in ① we get

$$\frac{du}{dy} = 2ay + b$$

$$0 = 2a\times 20 + b$$

$$0 = 40a + b$$

$$12 = 40a + 2b$$
$$0 = 40a + b$$

$$12 = b$$

$$\therefore a = \frac{-12}{40} = -0.3$$

$$u = -0.3y^2 + 12y + 0$$

$$\frac{du}{dy} = -0.6y + 12$$

$$\text{at } y=0 \ \left(\frac{du}{dy}\right)_{y=0} = 12$$

at $y=10$ $\left(\frac{du}{dy}\right)_{y=10} = 12-6=6$

at $y=20$ $\left(\frac{du}{dy}\right)_{y=20} = 12-12=0$

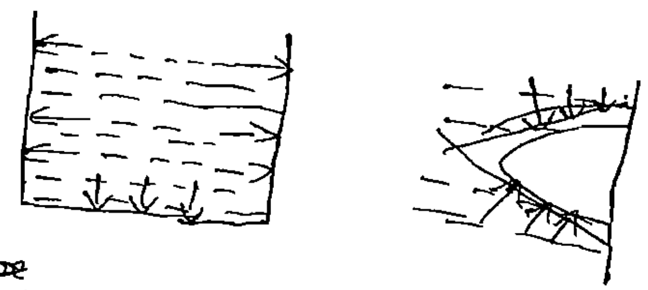
$\tau = \mu \cdot \frac{du}{dy}$

$y=0$ $\tau = 0.85 \times 12 = 10.2 \ N/m^2$

$y=10$ $\tau = 0.85 \times 6 = 5.1 \ N/m^2$

$y=20$ $\tau = 0.85 \times 0 = 0$

Fluid statics



when ever a liquid is contained in a venel the liquid will enert forces on the surfaces with which it comes into contact because of its own weight

The force enerted by the liquid on the surface with which it comes into contact is known as pressure. It is denoted by P unit is N or kN Dimensional Formula is MLT^{-2}

presure always acts normal to the surface of contact If it is acting on a curved surface is acts $\perp$r to the tangent at that point

Intensity of presure :-

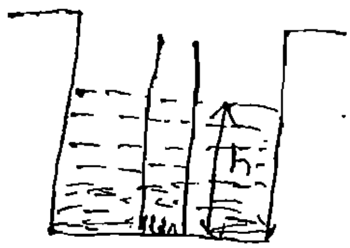
It is defined as the ratio b/n total presure and area of cross section on which it is acting.

units :- N/mm2 (or) N/m2 / (or) N/cm2

1 pascal = 1 N/m2

Presure head :-

Consider a venel containing some liquid as shown in figure. we know that the liquid will enert ~~pressure~~ prenure and all sides and bottom of the venel Now,



Let a cylinder be made to stand in the liquid as shown in figure

Let w = specific wt of liquid

h = height of liquid in cylinder

A = Area of the cylinder at base

A little consideration will show that there will be some pressure on cylinder base due to wt of the liquid in the cylinder

$\therefore$ presure at the base of the cylinder (P) =

$$= \frac{\text{wt of the liquid in the cylinder}}{\text{Area of the cylinder at base}}$$

$$\omega = \frac{W}{V}$$

$$\therefore W = \omega \cdot V$$

$$\therefore W = \omega . A \times h$$

weight of the liquid in the cylinder $(W) = \omega A \times h$

$$\therefore p = \frac{\omega . \cancel{A} \times h}{\cancel{A}}$$

$$\therefore p = \omega . h \quad (p = \rho . g h)$$

$$h = \frac{p}{\omega}$$

pressure head $(h) = \frac{p}{\omega}$

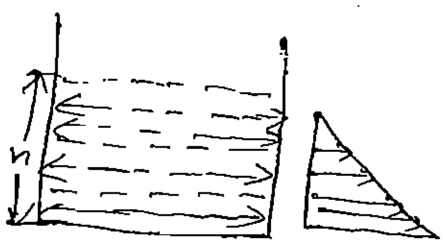
$\therefore$ pressure head is inversely proportional to the specific weight of liquid

pressure head changes with liquid

$$h = \frac{p}{\rho . g}$$

Units of pressure head is metres of liquid (kerosene, water, oil, mercury etc --)

Pressure diagram :-



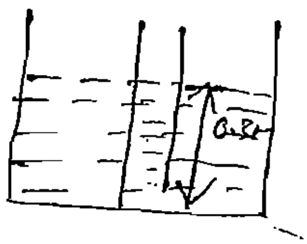
The graphical representation of the variation of ~~...~~ Intensity of pressure along with the depth of the fluid is know as pressure diagram.

generally the pressure diagram shape is triangle with "0" at top and maximum at the base

1) Calculate the pressure due to a column of 0.3m of A) water B) an oil of specific gravity is 0.8 and c) mercury of specific gravity 13.6 Take density of water as 1000

Sol/

$p = \rho.g.h$



water

$p = 1000 \times 9.81 \times 0.3 = 2943\ N/m^2 = \frac{2943}{10^4}$

$= 0.2943\ N/cm^2$

oil

$G = 0.8$

$\frac{\rho_{oil}}{\rho_{water}} = 0.8 \Rightarrow \rho_{oil} = 0.8 \times 1000 = 800\ kg/m^2$

$p = \rho.g.h = 800 \times 9.81 \times 0.3$

$= 2354\ N/m^2$ or $0.2354\ N/cm^2$

mercury

$G = 13.6$

$\frac{\rho_{Hg}}{\rho_{water}} = 13.6 \Rightarrow \rho_{Hg} = 13.6 \times 1000 = 13600\ kg/m^2$

$p = \rho.g.h = 1360 0 \times 9.81 \times 0.3 = 40025\ N/m^2$ or $4.0025\ N/cm^2$

2) The pressure intensity at a point in a fluid is given 3.924 N/cm^2 Find the corresponding height of the fluid when the fluid is (a) water (b) oil of G 0.9 c) mercury

Sol

$$p = w.h$$

$$p = \rho.g.h$$

$$p = 3.924\ N/cm^2 = 3.924 \times 10^4\ N/m^2$$

(a) water

$$h = \frac{p}{\rho.g} = \frac{3.924 \times 10^4}{1000 \times 9.81}$$

$h = 4m$ of water

(b) oil

$$G = 0.9$$

$$\frac{\rho_{oil}}{1000} = 0.9 \Rightarrow \rho_{oil} = 900\ kg/m^3$$

$$h = \frac{3.924 \times 10^4}{900 \times 9.81}$$

$h = 4.44$ m of oil

(c) Mercury

$$G = 13.6$$

$$\frac{\rho_{Hg}}{1000} = 13.6 \Rightarrow \rho_{Hg} = 13.6 \times 1000\ kg/m^3$$

$$h = \frac{3.924 \times 10^4}{13.6 \times 1000 \times 9.81}$$

$h = 0.29$ m of mercury

3) An oil of sp gra 0.9 is contained in a vessel at a point the ht of oil is 40m find the corresponding ht of water at that point.

sol:-

$\boxed{p = \cancel{\frac{\omega}{\rho_o g h}}}$ $p = \overset{\omega}{\rho_o} g\, h$

$p = (0.9 \times 1000) \cdot 9.81\,(40)$

$p = 353160\ N/m^2$

$h = \frac{p}{\rho \cdot g}$

$h = \frac{353160}{1000 \times 9.81} = 36m$ of water.

4) Convert a pressure head of 10 meters of water into a pressure head in m of (a) mercury (b) kerosene (c) carbon tetra chloride.

$p = \omega h$

$p = \rho_\omega g\, h$

case-1

$p_1 = \rho_{\omega_1} g\, h_1$

$p_1 = S_1 \rho_\omega g\, h_1 \longrightarrow (1)$

$S_m = \frac{\rho_\omega}{\rho_\omega}$

$\rho_m = S_m \rho_\omega$

case-2

$p_2 = \rho_{\omega_2} g\, h_2$

$\rho_\omega = S_1 \rho_\omega$

$p_2 = S_2 \rho_\omega g h_2 \longrightarrow (2)$

$S_2 = \frac{\rho_{\omega_2}}{\rho_\omega}$

$(1) = (2)$

$\rho_{\omega_2} = S_2 \rho_\omega$

$S_1 h_1 = S_2 h_2$

$h_w = 10m$

$S_1 h_1 = S_2 h_2$

$1 \times 10 = 13.6\, h_2$

$h_2 = 0.735$ m Hg

$S_1 h_1 = S_2 h_2$

$1 \times 10 = 0.8 \times h_2$

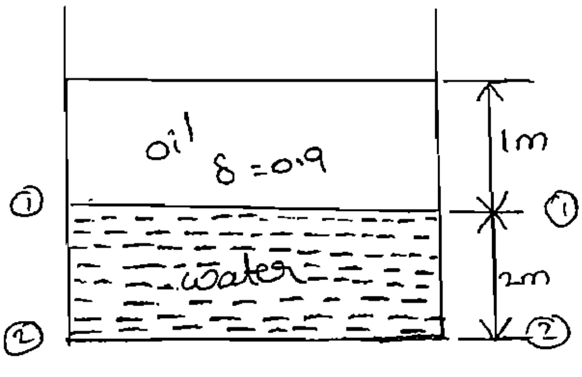
$\therefore h_2 = 12.5$ m of kerosene

$S_1 h_1 = S_2 h_2$

$1 \times 10 = 1.594 \times h_2$

$\therefore h = 6.273$ m of CCl_4

5) An open tank contain water upto a depth of 2m and above it an oil of spe gra 0.94 for a depth of 1m height. Find the pressure intensity (a) at the interface of the two liquids (b) at the bottom of the tank.



Sol:-

① - ①

$p = wh = \rho \cdot g \cdot h$

$= S_{oil} \times \rho_w \times g \cdot h$

$= 0.9 \times 1000 \times 9.81 \times 1$

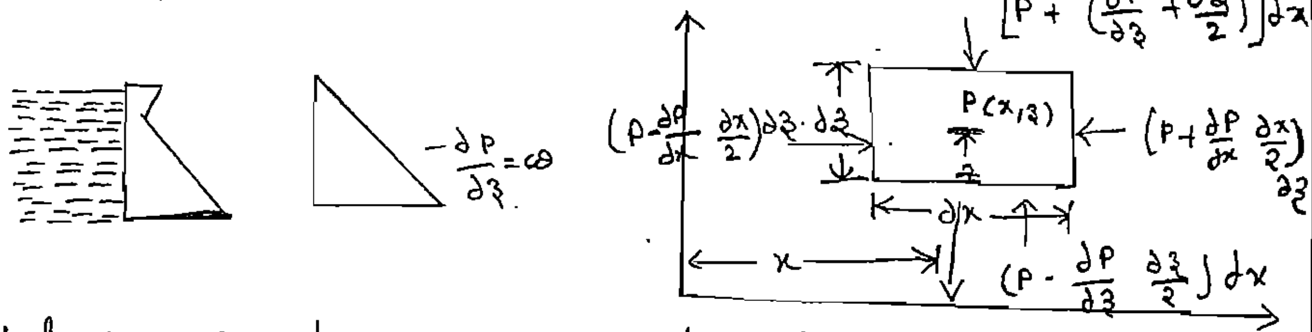
$= 8829\ N/m^2$

② - ②

$p = p_{oil} + p_{water}$

$= 0.8829 + \frac{1000 \times 9.81 \times 2}{10^4} = 2.844\ N/cm^2$

** Variation of static pressure:-



Let us consider a fluid element of size $\partial x, \partial z$ and of unit length as shown in figure

Let the static pressure at the centre of the element "o" be p The pressure is defined by the co-ordinates (x, z)

The static pressure force on the left face of the element $= \left(p - \frac{\partial p}{\partial x}\frac{\partial x}{2}\right)\partial z$

The static pressure force on the right face of the element $= \left(p + \frac{\partial p}{\partial x} \cdot \frac{\partial x}{2}\right)\partial z$

The static pressure force on the bottom face of the element $= \left(p - \frac{\partial p}{\partial z} \cdot \frac{\partial z}{2}\right)\partial x$

The static pressure force on the top face of the element $= \left(p + \frac{\partial p}{\partial z} \times \frac{\partial z}{2}\right)\partial x$

$\therefore$ weight of the fluid element $= [V \times \omega]$

$= (\partial x \cdot \partial z \cdot 1)\,\omega.$

$= \partial x\, \partial z\, \omega$

For static equilibrium the summation of force in x and z directions must be zero

i.e $\Sigma f_x = 0$ and $\Sigma f_z = 0$.

$$\Sigma F_x = 0$$

$\rightarrow +$ $\leftarrow -$ $\uparrow +$ $\downarrow -$

$$\left(p - \frac{\partial p}{\partial x} \frac{\partial x}{2}\right)\partial z - \left(p + \frac{\partial p}{\partial x} \cdot \frac{\partial x}{2}\right)\partial z = 0$$

$$-\cancel{2} \frac{\partial p}{\cancel{\partial x}} \cdot \frac{\cancel{\partial x}}{\cancel{2}} = 0$$

$$-\frac{\partial p}{\partial x} = 0$$

The above eq indicates that the pressure does not vary in x-direction. In other words the static pressure remains constant in the horizontal direction

$$\Sigma f_z = 0$$

$$-\left(p + \frac{\partial p}{\partial z} \frac{\partial z}{2}\right)\partial x + \left(p - \frac{\partial p}{\partial z} \frac{\partial z}{2}\right)\partial x - (\partial x \cdot \partial z)\, w = 0$$

$$-\left[2 \frac{\partial p}{\partial z} \frac{\partial z}{2}\right]\partial x$$

$$-\frac{\partial p}{\partial z} = w.$$

$$\boxed{-\frac{\partial p}{\partial z} = w}$$

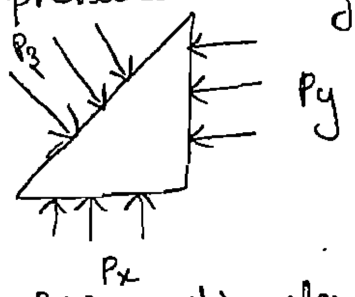
Hydrostatic Law:- The pressure at any point in a fluid at rest is obtained by the hydrostatic law. It states that the rate of ↑ce of pressure in vertically downward direction must be equal to the specific wt of the fluid at that point mathamatically it can be expressed as

$$-\frac{\partial p}{\partial z} = w.$$

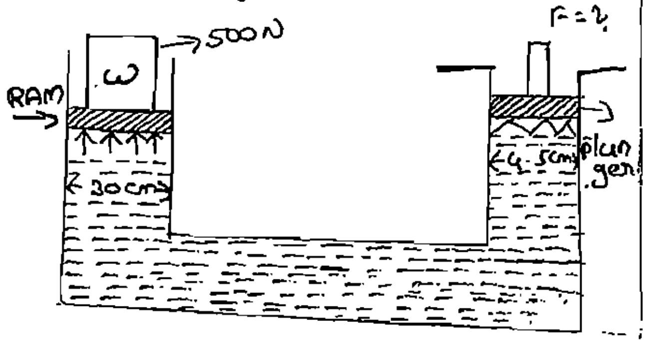
∴ –ve sign indicates the pressure ↓des in the direction in which z ↑ces in the ~~up~~upward direction

Pascal's law :-

It states that the intensity of pressure at any point in the fluid at rest same in all directions.



(1) A Hydrolic press has a ram of 30cm diameter and a plunger of 4.5 cm diameter. the wt lifted by the hydrolic press due to force applied by plunger is 500N find out the force applied on the plunger to lift 500N.



Sol :- Force on the plunger = pressure × Area of plunger.

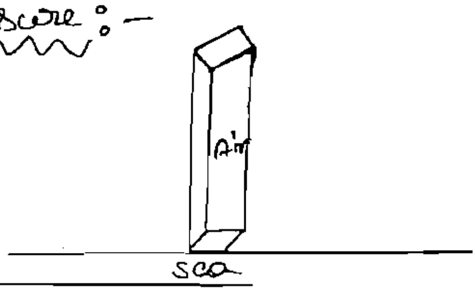
$$\text{pressure on fluid} = \frac{\text{weight}}{\text{Area}} = \frac{500}{\frac{\pi\left(\frac{30}{100}\right)^2}{4}} = 7073\ N/m^2$$

According to pascal's law pressure acting on the Ram is equal to the pressure acting on the plunger.

$$\therefore \text{Force on the plunger} = 7073 \times \frac{\pi \times \left(\frac{4.5}{100}\right)^2}{4}$$

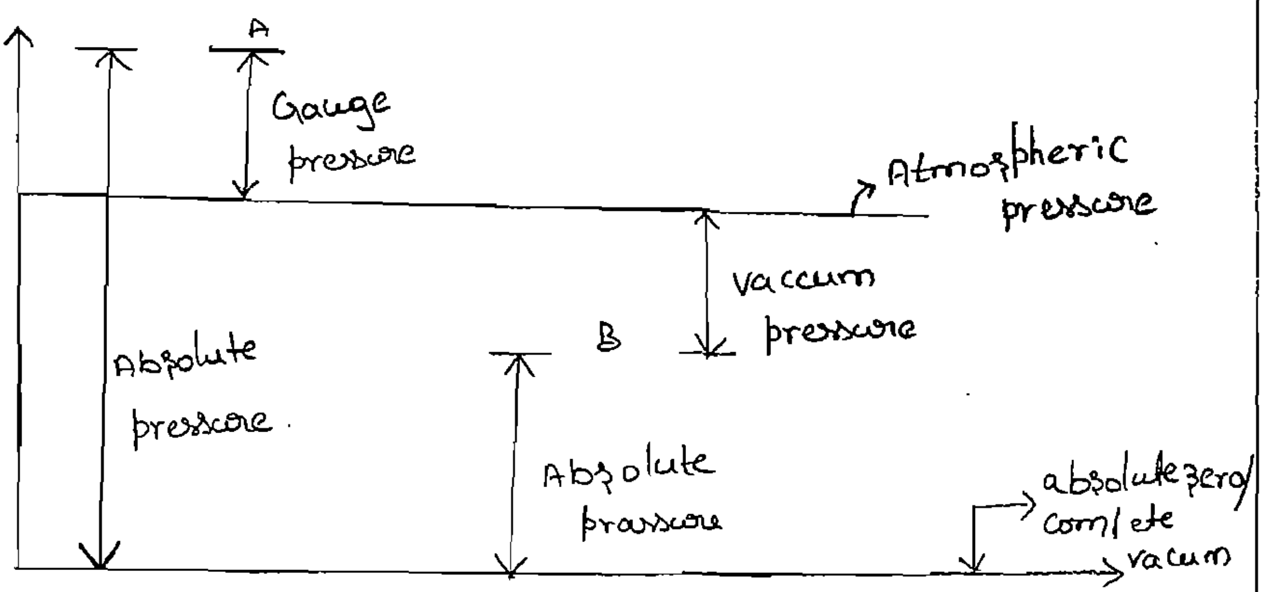
$$= 11.25\ N$$

Atmospheric pressure :-



Air posses some wt hence it applies some pressure on the surface of the fluid. The pressure exerited by the air because of its own wt on the surface of the fluid is known as atmospheric pressure. The wt of the air is depends upon (1) Height of the atmosphere (2) Temperature. and (3) Humidity

Since it is compressible. It also changes from place to place. Hence standard atmospheric pressure is taken as the pressure exerted by the column of air of 1sq.cm. cross sectional area and wt is equal to the atmosphere is 1.03 kg. Hence the standard atmospheric pressure at sea level is taken as 1.03 kg/cm². It can also be expressed as 10.3 m of water /76 cm of mercury. The pressure caused by atmosphere is measured by using barometer. Hence atmospheric pressure also called as Barometric pressure.



The relationship b/n absolute pressure, gauge pressure and vaccum pressure are shown in figure and mathematically can be expressed as

Absolute pressure = Atmospheric pressure + G.P

= Atmos. P - Vacum pressure

→ convert the following gauge pressure into absolute pressure.

standard Atm.Pre

* 34 cm of Hg → 76 cm of Hg = 110 cm Hg.
* 5m of water → 10.34 m of water = 15.34 m of water.
* 0.666 kg-f/cm^2 → 1.034 kg/cm^2 = 1.7 kg/cm^2
* 0.25 N/mm^2 → 0.101 N/mm^2 = 0.3514 N/mm^2
* 50 kN/m^2 → 101.4 kN/m^2 = 151.4 kN/m^2

→ convert the following vaccum pressure into the absolute pressures.

* 2.6 cm of Hg = 50 cm of Hg.
* 5.34 m of water = 5 m of water

$$\left.\begin{array}{l} 1.03\ kg/cm^2 \\ 0.1014\ N/mm^2 \\ 101.4\ kN/m^2 \\ 76\ cm\ of\ mercury \\ 10.34\ m\ of\ water. \end{array}\right\}\ \text{standard atmospheric pressure.}$$

Pressure measurement:-

* Absolute pressure zero (or) complex vaccum pressure (or) Absolute pressure.

* Gauge pressure

* vaccum pressure.

The pressure on a fluid is measured in two different systems.

In 1st system, it is measured above the absolute 0 it is called absolute zero (or) comlete vaccum and it is Absolute pressure. pressure / complex vaccum pressure / Absolute pressure.

In 2nd system the pressure is measured by taking atmospheric pressure as reference. Then it is known as change pressure / vacume pressure

If the pressure is measured above the atmospheric pressure then it is known as Gauge pressure

If the pressure is measured below the atmospheric pressure then it is known as vaccum pressure.

* 0.034 kg/cm2 = 1 kg/cm2
* 0.05 N/mm2 = 0.0514 N/mm2
* 51.4 kN/m2 = 50 kN/m2

* The only instrument by which we get absolute pressure at a point is known as ANAROID BAROMETER.

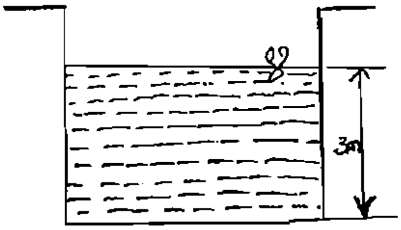
(1) What are the gauge pressure and absolute pressure at a point 3m below the free surface of a liquid having a density of $1.53 \times 10^3\ kg/m^3$ if the Atmos pres is equivalent to 750mm of Hg

Sol:- Gauge pressure $= wh$

$= \rho \cdot g \cdot h$

$= (1.53 \times 10^3)\ 9.81 \times 3$

$= 45027.9\ N/m^2$



Atmospheric pressure $= \rho \times g \cdot h$

$= (13.6 \times 1000) \times 9.81 \times 0.75$

$= 100062\ N/m^2$

Absolute pressure $= G.P + A.P$

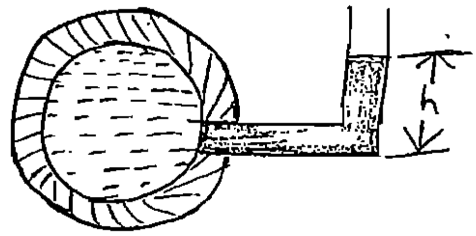
$= 145.08\ kN/m^2$

Pressure measuring devices:-

- Pressure measuring devices.
  - Mano meters.
    - Simple manometers.
      - Piezometer.
      - U-tube manometer
      - Single column m.m
        - Vertical single column m.m
        - Inclined single column m.m
    - Differential manometers
      - U-tube m.m
      - Inverted U-tube m.m.
    - Micro manometers
  - Mechanical pressure gauges.

mechanical pressure gauges
→ Diaphram p.G.
→ Bourden tube pressure gauges.
→ Dead wt p.G.
→ Bellows p.G.

manometers :-



manometers are defined as the devices used for measuring the pressure at a point in the fluid by balancing the column of the fluid by the same or another column of the fluid.

These are classified as

(1) simple manometers
(2) Differential manometers.

and

(3) micro manometers.

1) Simple manometer :-

A simple manometer consists of a glass tube having one of its ends connected to a point where the pressure is to be measured and other end remains open to atmosphere

Common types of simple m.m are

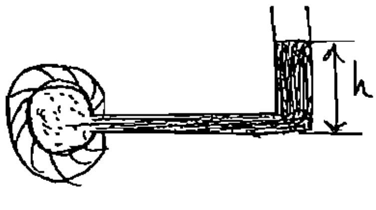
i) Piezo m.m
ii) U-tube m.m
iii) Single column m.m

Single column m.m is divided into

a) Vertical single column m.m
b) Inclined single column m.m

1) Piezo manometer :-

It is the simplest form of m.m used for measuring ~~(measurement)~~ gauge pressure one end of this m.m is connected to the point where the pressure is to be measured and other end is open to the atmosphere as shown in fig. The rise of liquid gives the pressure head at that point.



$$\text{pressure} = w \cdot h$$
$$= \rho \cdot g \cdot h$$

where, ρ = density of manometric liquid

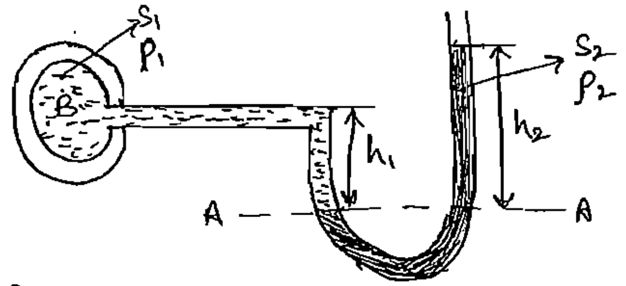
h = piezometric height

Limitations :-

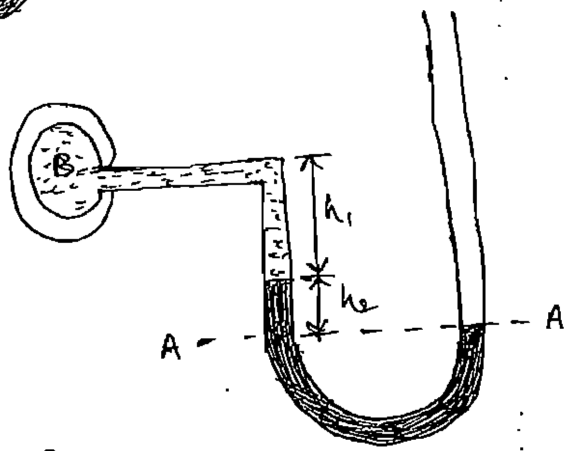
1) It cannot be used to find out moderate or low positive pressure.

2) It cannot be used to measure pressure of gases.

(ii) U-tube manometer :-



For gauge pressure.



For vacuum pressure.

A U-tube manometer consists of a glass tube bend in U-shape. One end of the manometer connected to a point where pressure is to be measured other end is opened to the atmosphere. It consists of Hg as manometric liquid or any other liquid whose specific gravity is greater than the specific gravity of the liquid whose pressure is to be measured.

Use: It is used to find out high +ve pressures / -ve pressures at a particular point in a pipe-line limitation :

It cannot be used to find out the low pressures

Points to remember :-

1) The liquid used as manometric liquid should have high specific weight and very low vapour pressure

2) Mercury possess high specific weight and low vapour pressure thats why Hg is used as manometric liquid

3) Whatever may be the manometric liquid and whatever may be the liquid flowing through the pipe. All the pressure heads shall be expressed in terms of meters of water

4) The lowest mercury level shall be considered as datum line

5) A two points in the manometer on the datum line the pressure in the left limb is equal to pressure in the right limb

For Gauge pressure :-

let B is the point at which the pressure is to be measured whose value is p. The datum line is AA.

let h_1 = height of the liquid above the datum line and

h_2 = ht of heavy liquid above the datum ~~liquid~~ line

S_1 = specific gravity of the light liquid

ρ_1 = density of the light liquid

S_2 = specific gravity of heavy liquid

ρ_2 = density of heavy liquid

pressure above the datum line AA on the left limb

$= p + \rho_1 g h_1$ $\quad \boxed{\begin{aligned} p &= w \cdot h \\ &= \rho \cdot g \cdot h \end{aligned}}$

$= p + S_1 \times 1000 \times g \cdot h_1$

pressure on the datum line AA on the right limb

$= \rho_2 g h_2$

$= S_2 \times 1000 \times g \cdot h_2$

They are equal

$p + S_1 \times 1000 \cdot g \cdot h_1 = S_2 \times 1000 \cdot g \cdot h_2$

$\therefore p = 1000 g [S_2 h_2 - S_1 h_1]$

For vacuum pressure:

Pressure on right limb $= 0$

left limb $= p + \rho_1 g h_1 + \rho_2 g h_2$

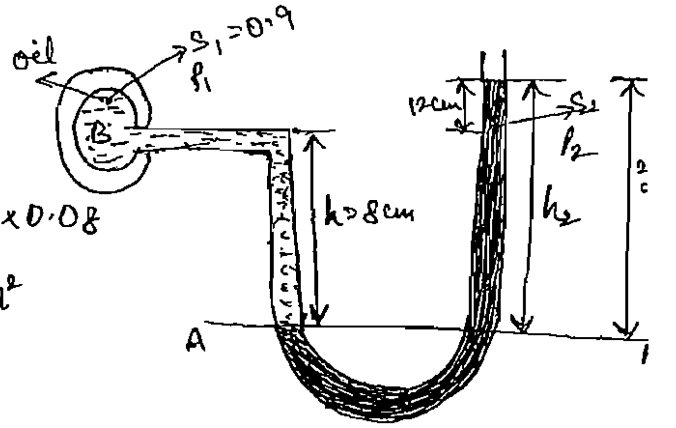
$\therefore p + \rho_1 g h_1 + \rho_2 g h_2 = 0$

$\therefore p = -[\rho_2 g h_2 + \rho_1 g h_1]$

$p = -[S_2 \cdot 1000\, g \cdot h_2 + S_1 \cdot 1000 \cdot g \cdot h_1]$

$\therefore p = -1000 \cdot g [S_2 h_2 + S_1 h_1]$

1) The right limb of a simple U-tube mm containing Hg is opened to the atmosphere while the left limb is connected to a pipe in which a fluid of specific gravity 0.9 is flowing. The center of pipe is 12 cm below the level of Hg in right limb. Find the pressure of the fluid in the pipe if the difference of Hg level in the two limbs is 20cm.



Sol: left limb $= P_B + \rho_1 \times g \times h_1$

$= P_B + 0.9 \times 1000 \times 9.81 \times 0.08$

$= P_B + 706.32 \; N/m^2$

Right limb $= \rho_2 g h_2$

$= 13.6 \times 1000 \times 9.81 \times 0.2$

$= 26683.2 \; N/m^2$

$P_B + 706.32 = 26683.2$

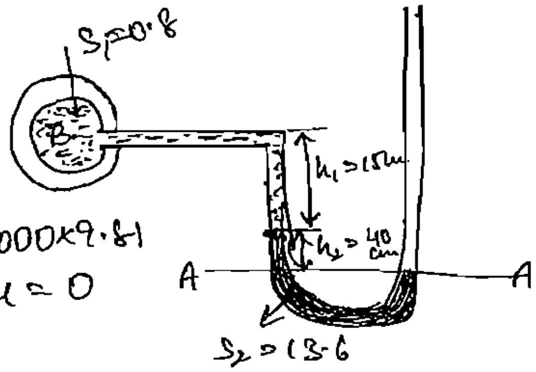
$P_B = 26683.2 - 706.32$

$= 25976.88 \; N/m^2$

$= 2.59 \; N/cm^2$

2) A simple U-tube manometer containing Hg is connected to a pipe in which a fluid of specific gravity 0.8 and having vacuum pressure is flowing. The other end of the manometer is opened to the atmosphere. Find the vacuum pressure in the pipe if the difference of Hg level in the two limbs is 40 cm. and height of fluid in the left limb from the center of pipe is 15 cm. below

Sol: left limb = $P_B + \rho_1 g h_1 + \rho_2 g h_2$



Right limb $= 0$

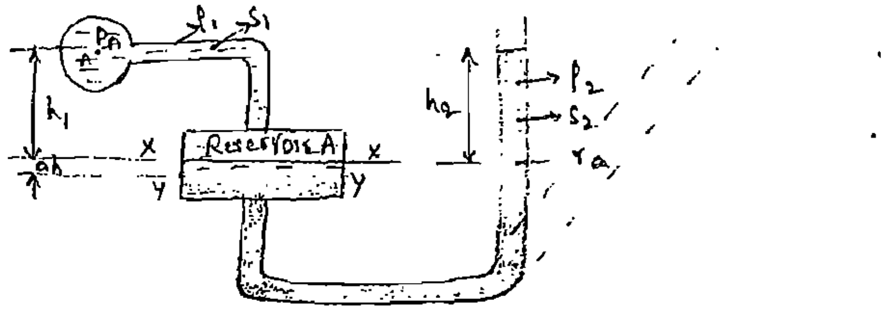
$\therefore P_B + 0.8 \times 1000 \times 9.81 \times 0.15 + 13.6 \times 1000 \times 9.81 \times 0.4 = 0$

$\therefore P_B = -[54543.6]\ N/m^2$

$P_B = -5.45\ N/cm^2$

-ve sign indicates it is a vacuum pressure

iii) Single column Manometer:-



A single column Manometer is a Modified form of a U-tube Manometer in which a Reservoir having a large cross sectional area (about 100 times) as compared to the area of the tube is connected to one of the limbs say a left limb of Manometer as shown in figure.

Due to a large cross sectional Area of a Reservoir, for any Variation in pressure the change in the liquid level to the Reservoir will be Very Small which may be Neglected and hence the pressure is given by the height of the liquid in the other limb. The other limb May be Vertical or inclined. Hence there are two types of single column Manometer. They are a) Vertical single column MM & b) Inclined single column MM.

a) Vertical Single column Manometer:-

The figure shows the Vertical single column Manometer. Let XX be the datum line in the Reservoir and in the Right limb of the Manometer. When it is not connected to the pipe due to high pressure at A, the heavy liquid in Reservoir will be pushed downward and there will be rise in heavy liquid level in right limb.

Let Δh = Fall of heavy liquid in the Reservoir.

h_2 = Rise of heavy liquid in the Right limb

h_1 = height of the centre of the pipe above the datum line XX.

P_A = pressure at the A which is to be Measured.

A = cross sectional Area of the Reservoir

a = cross sectional area of the Right limb.

S_1 = Specific gravity of the liquid in the pipe line.

S_2 = Specific gravity of heavy liquid in the Reservoir on the Right limb

ρ_1 = density of liquid in the pipe

ρ_2 = density of liquid in the Reservoir.

Y–Y = Datum line after mm connected to the pipe.

Fall of heavy liquid in the Reservoir will cause a rise of heavy liquid in the Right limb

$$A \times \Delta h = a \times h_2$$

$$\Delta h = \frac{a}{A} \cdot h_2$$

pressure in the Right limb $= \rho_2 \cdot g\,(h_2+\Delta h)$

pressure in the left limb $= P_A + (\rho_1 g\,(h_1+\Delta h))$

$$P_A + \rho_1 g\,(h_1+\Delta h) = \rho_2 g\,(h_2+\Delta h)$$

$$P_A = \rho_2 g(h_2+\Delta h) - \rho_1 g\,(h_1+\Delta h)$$

$$= \rho_2 g h_2 + \rho_2 g\,\Delta h - \rho_1 g h_1 - \rho_1 g\,\Delta h$$

$$= \Delta h \cdot g(\rho_2 - \rho_1) + g(\rho_2 h_2 - \rho_1 h_1)$$

$$= \frac{a}{A} \cdot h_2 \cdot g\,(\rho_2 - \rho_1) + g\,(\rho_2 h_2 - \rho_1 h_1)$$

In the above Equation, it is clear that the ratio of $\frac{a}{A}$ becomes Very small and can be neglected

$$\therefore P_A = g\,(\rho_2 h_2 - \rho_1 h_1)$$

In the above Equation h_1 is known and hence by knowing h_2 the pressure at A can be calculated.

* problems

1) A single column mm is connected to a pipe containing a liquid of specific gravity 0.9 as shown in figure. Find the pressure in the pipe if the area of the Reservoir is 100 times area of the tube for the Manometric Reading shown in the figure. the specific gravity of a Hg is 13.6.

Sol:-

In the figure

$h_1 = 20cm$; $h_2 = 40cm$

$S_A = 0.9$; $S_m = 13.6$

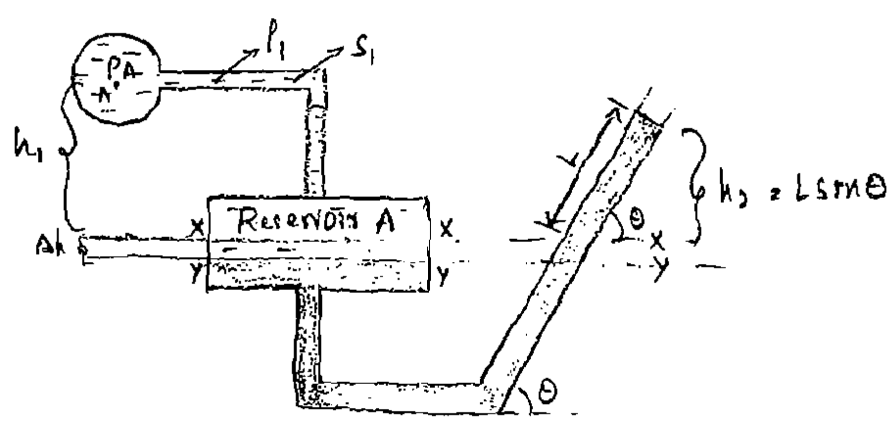
$$\therefore p = \left[13.6 \times 1000 \times 9.81 \times \left(\frac{40}{100}\right) - \left(0.9 \times 1000 \times 9.81 \times \frac{20}{100}\right)\right]$$

$= 51600.6 \ N/m^2$

$= 5.16 \ N/cm^2$

($\because$ by considering $\frac{a}{A}$ term also we get $p = 5.21 \ N/cm^2$)

b) Inclined single column Manometer:-



This Manometry is more sensitive due to inclination in the distance. Moved by the heavy liquid in the Right limb will be more.

let L = Length of the heavy liquid moved in the Right limb from x-x

θ = Inclination of the Right limb with the horizontal.

h_2 = Vertical rise of heavy liquid in the Right limb from x-x
$= L\sin\theta$

pressure in the pipe line $A = \rho_2 \cdot g \cdot L\sin\theta - \rho_1 g h_1$

2) Differential Manometer :-

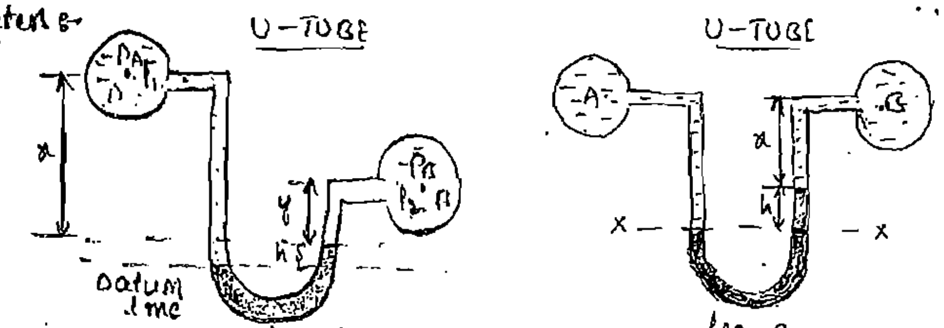


fig-A fig-B

Differential Manometer are devices used for Measuring the difference of pressure between two points & in two different pipes which are in the same level / different levels.

A differential M.M consists of U-tube containing heavy liquid whose two ends are connected to the two points whose difference of pressure is to be Measured. Most commonly used Differential M.M's are:

i) U-tube differential M.M

ii) Inverted U-tube differential M.M.

i) a) U-tube differential M.M at two different levels :- (fig-A) :-

Let the two points A and B are at different levels and also contains liquids of different specific gravity. These 2 points are connected to the U-tube differential Manometer.

Let the pressure at A and B are p_A and p_B

h = difference of Mercury level in the U-tube

y = distance of centre of B from the Mercury level in the Right limb.

x = distance of the centre of the A from the Mercury level in the left limb.

ρ_1 = density of the liquid at A

ρ_2 = density of the liquid at B

ρ_m = (spec) density of the heavy liquid
↓
Manometric

Above the datum line pressure in the left limb is Equal to pressure in the Right limb.

pressure above the datum line in the left limb

$$= P_A + \rho_1 g(h+x) \quad \text{--- (1)}$$

pressure above the datum line in the Right limb

$$= P_B + \rho_2 g y + \rho_m g h \quad \text{--- (2)}$$

$$(1) = (2)$$

$$P_A + \rho_1 g(h+x) = P_B + \rho_2 g y + \rho_m g h$$

$$P_A - P_B = \rho_2 g y + \rho_m g h - \rho_1 g(h+x)$$

$$= g(\rho_2 y + \rho_m h - \rho_1 h - \rho_1 x)$$

$$= hg(\rho_m - \rho_1) + \rho_2 g y - \rho_1 g x$$

b) <u>U-tube differential</u> M.M <u>at the same level</u> :- fig (B) :-

Left limb $= P_A + \rho_1 \cdot g(h+x)$

Right limb $= P_B + \rho_2 \cdot g x + \rho_m \cdot g h$

$$P_A - P_B = \rho_2 g x + \rho_m g h - \rho_1 g(h+x)$$

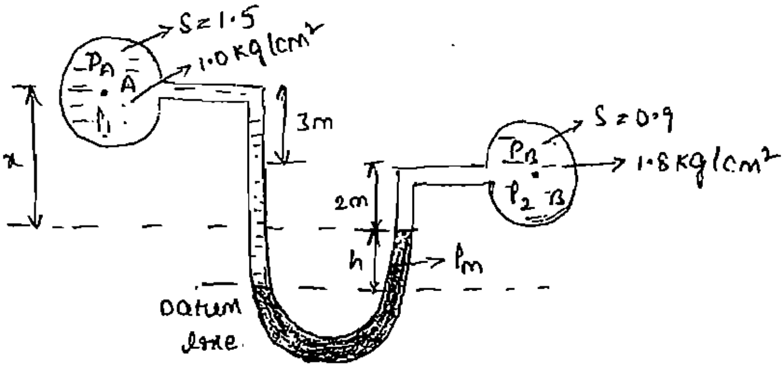
$= gh(\rho_m - \rho_1) + gx(\rho_2 - \rho_1)$ $\quad [\because \rho_2 = \rho_1]$

$= gh(\rho_m - \rho_1)$ $\quad$ [∵ They are in same line] having same liquid.

* problems

1) A differential M.M is connected at the two points A and B of a pipe as shown in figure. the pipe A contains a liquid of specific gravity of 1.5. while pipe B contains a liquid of specific gravity 0.9. the pressures at A and B are 1kg/cm² and 1.80 kg/cm² respectively. find the difference in Hg level in the differential MM.

Sol:



pressure in the left limb $= 1\times10^4 + 1.5\times1000\times9.81(3+2+h)$

$= 10^4 + 14715(5+h)$

pressure in the right limb $= 1.8\times10^4 + 0.9\times1000\times9.81\times2 + 13.6\times1000\times9.81h$

$= 35658 + 133416h$

Equating them we get

$10^4 + 14715(5+h) = 35658 + 133416h$

$14715(5+h) - 133416h = 25658$

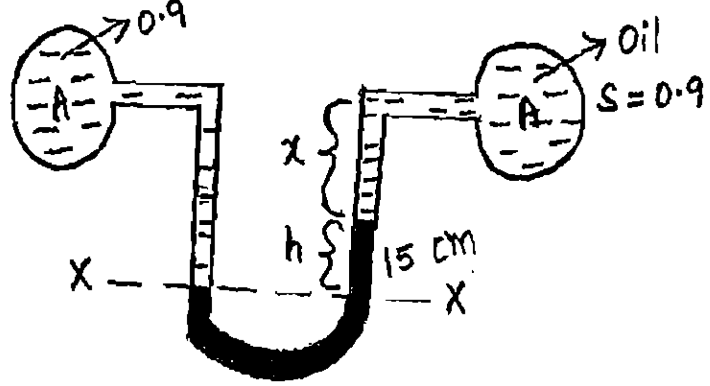
$73575 + 14715h - 133416h = 25658$

$47917 = (133416 - 14715)h$

$47917 = 118701 \cdot h$

$h = 0.4m = 40cm$

2) A pipe contains an oil of Specific gravity 0.9. A differential mm is Connected at the two points A and B Shows a difference in Hg level as 15 cm. Find the difference of pressure at 2 points.



$$P_A - P_B = gh(\rho_m - \rho_1)$$

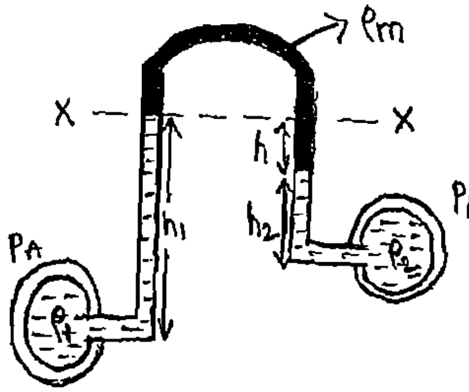
$$= 9.81 \times \frac{15}{100} \times (13.6 \times 1000 - 0.9 \times 1000)$$

$$= 1.47 (12.7) 1000$$

$$= 18669 \ N/m^2$$

$$= 18.66 \ KN/m^2$$

(ii) Inverted U-tube differential manometer:



It consists of a inverted U-tube containing a light liquid. The two ends of the U-tube are connected to the points whose difference of the pressure is to be measured. It is used for measuring low pressures.

Points to be remember:

1) The specific gravity of manometric liquid shall be less than specific gravity of liquids flowing through the two points.
2) The highest manometric liquid surface shall be considered as the datum line.
3) The pressure heads are to be determined below the datum line.

The manometer shown in figure connected to the points A and B. Let the pressure at A is more than the pressure at B.

Let

h_1 = height of the liquid in left limb below the datum line.

h_2 = height of the liquid in the Right limb

h = difference of light liquid

ρ_1 = density of liquid A

ρ_2 = density of liquid B

ρ_m = density of manometric liquid

p_A = pressure at point A

p_B = pressure at point B

Taking x-x as datum line the pressure in the left limb below the x-x is

left limb = $p_A - \rho_1 g h_1$

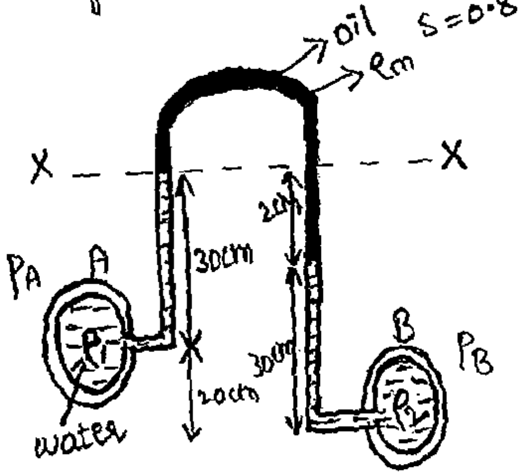
Right limb = $p_B - \rho_2 g h_2 - \rho_m g h$

We know that below the datum line the pressure in the left limb = the pressure in the Right limb.

$$p_A - \rho_1 g h_1 = p_B - \rho_2 g h_2 - \rho_m g h$$

$$\therefore \quad p_A - p_B = \rho_1 g h_1 - \rho_2 g h_2 - \rho_m g h$$

1) An inverted Differential manometer is Connected to two pipes A & B. which Convey water. The fluid in manometries oil of specific gravity 0.8 but the manometer readings Shown in figure. Find the pressure difference between A & B.



Left limb $= P_A - \rho_1 g h_1$

$$= P_A - 1000 \times 9.81 \times \frac{30}{100}$$

$$= P_A - 2943$$

Right limb $= P_B - \rho_2 g h_2 - \rho_m g h$

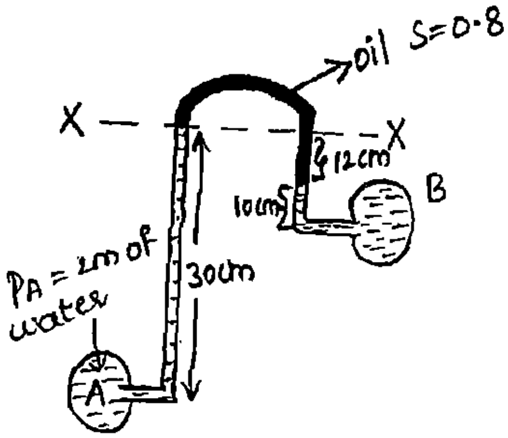
$$= P_B - 1000 \times 9.81 \times \frac{30}{100} - 0.8 \times 1000 \times 9.81 \times \frac{20}{100}$$

$$= P_B - 2943 - 156.96$$

$$P_A - P_B = -1569.6 \ N/m^2$$

$$\therefore \quad P_B - P_A = 1569.6 \ N/m^2$$

2) Water is flowing through 2 different pipes to which an inverted Differential manometer having an oil of Specific gravity 0·8 is connected. The pressure head in the pipe A is 2 mts of water. Find the pressure in the pipe B for the manometer Readings as shown in figure.



Sol:- $P_A = \rho g h$

$= 1000(9.81)2$

$= 19620 \ N/m^2$

pressure in the left limb

$= P_A - \rho g h_1$

$= 19620 - 1000(9.81)\frac{30}{100}$

$\Rightarrow 19620 - 2943 = 16677$

Right limb $= P_B - 1000(9.81)\frac{10}{100} - 0.8 \times 1000(9.81)\frac{12}{100}$

$= P_B - 981 - 941.76$

$= P_B - 1922.76$

$\therefore P_B = 16677 + 1922.76$

$= 18599.76 \ N/m^2$

$P_B = 1.8599 \ N/cm^2$

Micromanometer :

When an additional gauge liquid is used in a U-tube manometer, a large difference in meniscus leve may be obtained for a very small pressure differen

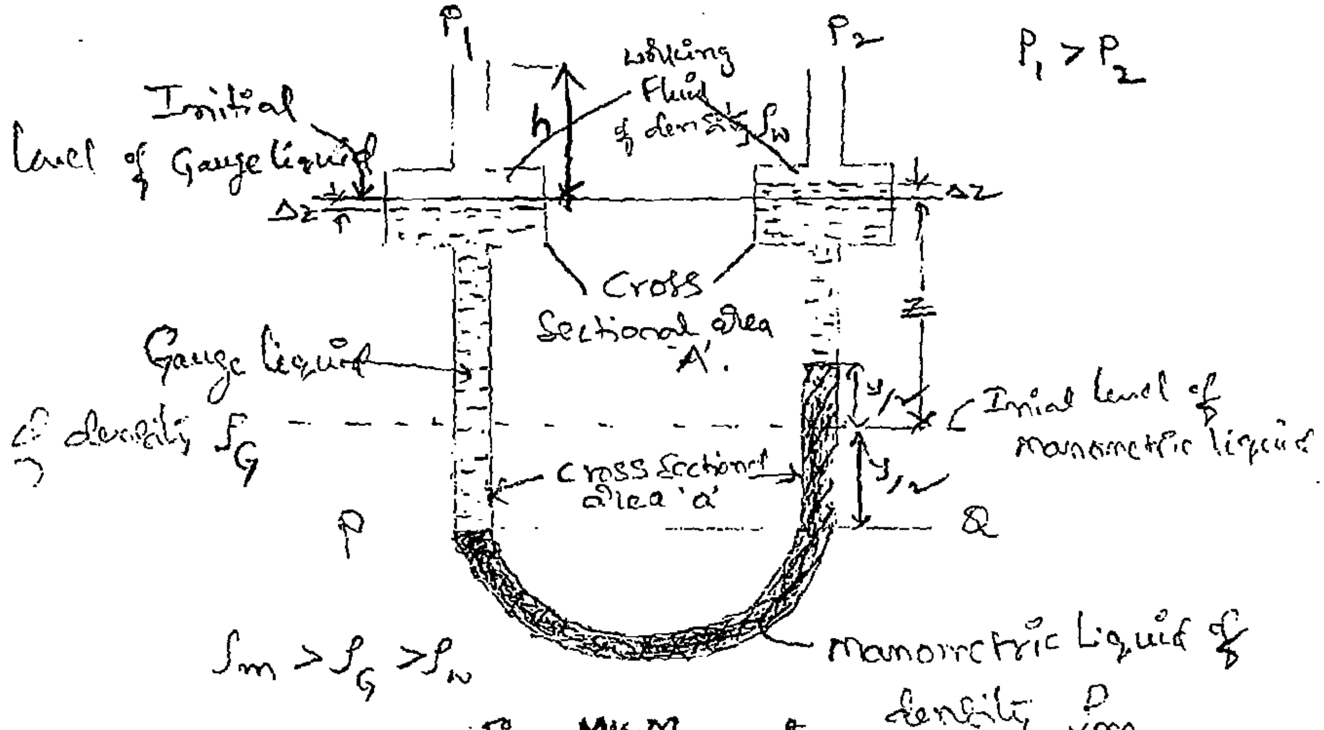


Fig. Manometer

The equation of hydrostatic equilibrium at PQ can be written as

$$P_1 + \rho_w g (h + \Delta z) + \rho_G\, g \left(z - \Delta z + \frac{y}{2}\right) = P_2 + \rho_w g (h - \Delta z) + \rho_G\, g \left(z + \Delta z - \frac{y}{2}\right) + \rho_m\, g y$$

where ρ_w, ρ_G and ρ_m are the densities of working fluid, gauge liquid and manometric liquid respecti from continuity of gauge liquid,

$$A \cdot \Delta z = a \cdot \frac{y}{2}$$

$$P_1 - P_2 = g y \left(\rho_m - \rho_G \left(1 - \frac{a}{A}\right) - \rho_w \frac{a}{A} \right)$$

If 'a' is very small compared to 'A'.

$$\therefore P_1 - P_2 \approx (\rho_m - \rho_G)\, g y$$

with a suitable choice for the manometric and gauge liquids so that their densities are close ($\rho_m \approx \rho_G$) a reasonable value of 'y' may be achieved for a small pressure difference,

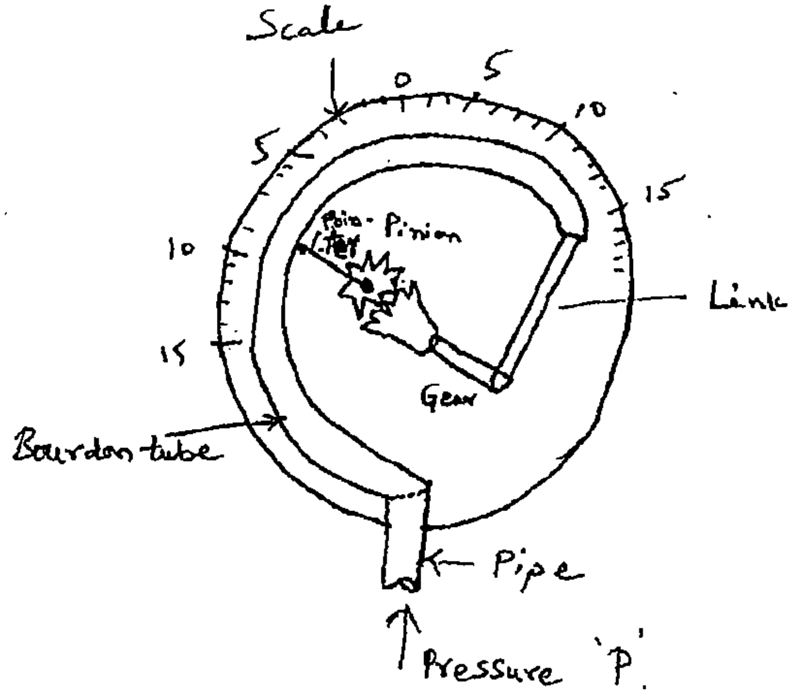
Pressure Gauges (or) Mechanical Gauges :

Whenever a very high fluid Pressure is to be measured a mechanical gauges is best suited for purpose. Because tube gauges (i.e. Manometers) cannot be conveniently used.

The following four are Mechanical gauges (or) Pressure gauges are used.

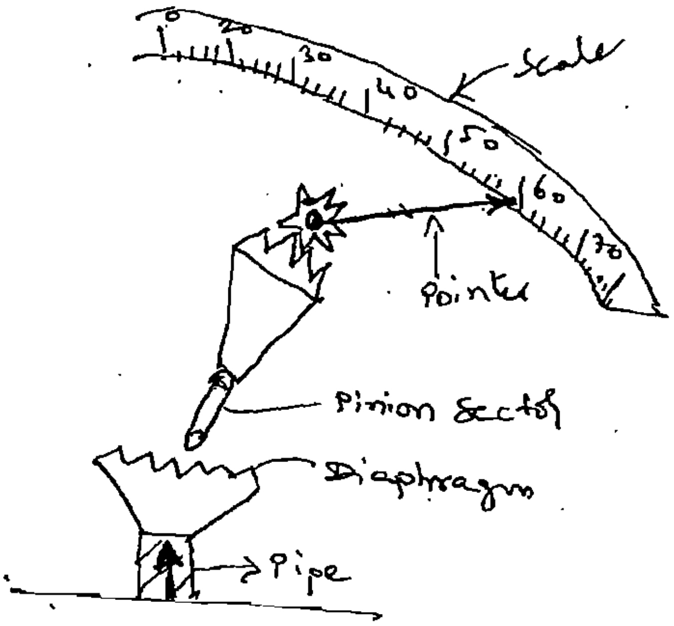
1) Bourdon's tube Pressure gauge
2) Diaphragm Pressure gauge
3) Dead Weight Pressure Gauge
4) Bellows Pressure Gauge.

1. Bourdon Pressure gauge



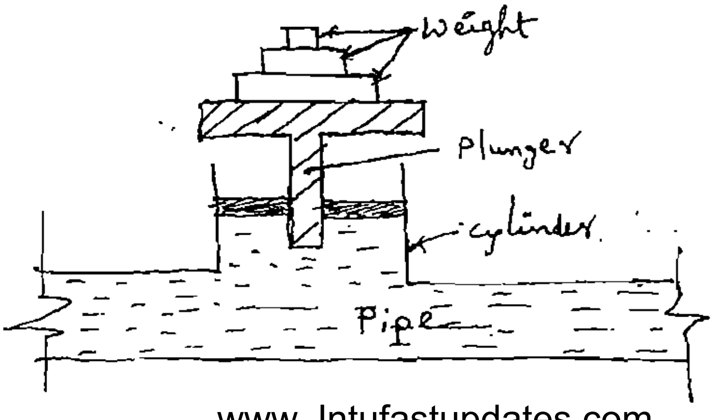
It consists of metal tube of elliptical cross section the tube is bent into a circular shape, One end of the tube is fixed & Other is free to move inward & Outward. There is a steel pointer is mounted on pinion with gear & link. The movement of the tube moves the link, the link moves the gear & Gear moves the pinion & the pinion moves the Pointer. The unit of scale is bar & N/m^2. The Pressure above / below the atmospheric pressure easily measure with help of bourdon's tube Pressure gauge.

2) Diaphragm Pressure gauge:



The Diaphragm pressure gauge consists of a corrugated diaphragm instead of Bourdon's tube. When the gauge is connected to fluid pipe. The fluid under pressure causes some deformation of the diaphragm. With the help of pinion arrangement, the elastic deformation of the diaphragm rotates the pointer. This pointer moves over a calibrated scale which directly gives the pressure as shown in figure.

3) Dead Weight Pressure Gauge:



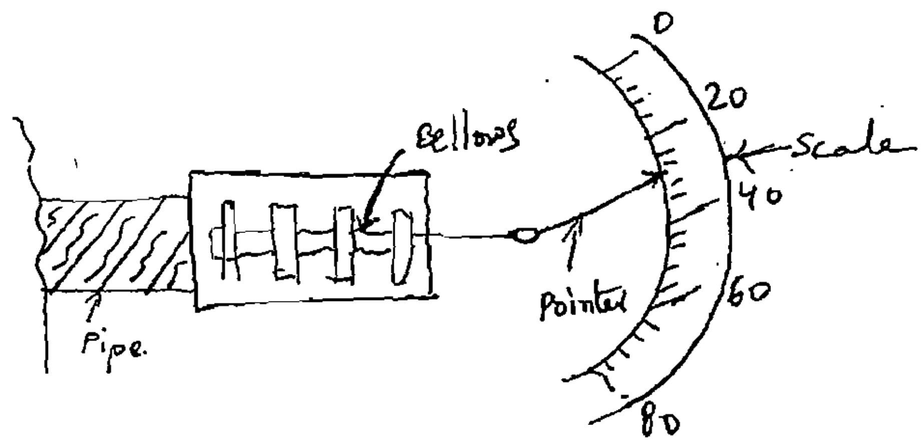
$$\text{Pressure} = \frac{\text{Load}}{\text{Area of Plunger}}$$

It consists of a plunger which can slide within a vertical cylinder as shown in fig. The fluid under press enters the cylinder & exerts a force on the plunger. Then Weights are placed on the top of the plunger to balance this force. The pressure intensity of the fluid is given by

$$\frac{\text{Weight placed on the top of the plunger}}{\text{Area of the plunger}}$$

Dead Weight pressure gauges are frequently used for calibrating other types of gauges.

4) Bellow's Pressure gauge:



It consists of a thin metallic tube having deep circumferential curvations called Bellows. As the pressure changes these Bellows expands/contracts and cause movement of pointer on a graduated circular disk as shown in the figure.

CIVIL Fluid Mechanics UNIT-I

TUTORIAL - I

→ Fluid Mechanics is the branch of science which deals with beha-viour of Fluids [Gases & Liquids]

→ Fluid Mechanics consists of :-

i) Fluid statics → It is the study of Fluids at rest.

ii) Fluid kinematics → It is the study of Fluids in motion neglecting Pressure force

iii) Fluid Dynamics → It is the study of Fluids in motion considering Pr. force

* Properties of Fluids :-

1) Density & Mass Density :- It is defined as the ratio of mass of fluid to the volume of fluid.

Mathematically,

$$\text{Density } (\rho) = \frac{\text{mass of fluid (kg)}}{\text{Volume of fluid } (m^3)}$$

$$\boxed{\therefore \rho = \frac{m}{V}} \quad kg/m^3.$$

$\therefore$ Density of water $= 1000\, kg/m^3$.

2) Weight Density :- It is defined as the ratio of weight of fluid to the volume of fluid.

Mathematically,

$$\text{Weight Density } (w) = \frac{\text{weight of fluid } (W)}{\text{Volume of Fluid } (V)}$$

$$\boxed{\therefore w = \frac{W}{V}} \quad \frac{N}{m^3} \;\&\; \frac{kN}{m^3}.$$

$$\therefore w = \frac{m \times g}{V} = \rho g \qquad \boxed{\therefore w = \rho g}$$ It is also called Sp. weight

$\therefore$ Weight Density of Water, $w = 1000 \times 9.81 = 9810\,N$

3. Specific Volume: It is defined as the ratio of Volume per unit mass.

Mathematically,

$$\text{Sp. Volume}(\vartheta) = \frac{\text{Volume of Fluid}(V)}{\text{mass of fluid}(m)}$$

$$\boxed{\therefore \vartheta = \frac{V}{m}} \quad \frac{m^3}{kg}$$

$$\boxed{\therefore \vartheta = \frac{1}{\rho}}$$ Reciprocal of Mass Density.

4. Specific Gravity:- It is defined as the ratio of weight density of liquid to the weight density of standard liquid (water)

g is same so cancelled

Mathematically,

$$\text{Sp. gravity}(S) = \frac{\text{weight density of liquid}}{\text{Weight density of Water}} = \frac{\rho_{liquid} \times g}{\rho_{water} \times g}$$

$[\because w = \rho g]$

Also, $$S = \frac{\text{mass density of liquid}}{\text{mass density of water}}$$

* 's' for mercury = 13.6
* 's' for water = 1.

Problems on Properties of Fluids

Q, Calculate Specific Weight, Density & Specific gravity of 1 of a liquid which weighs 7 N.

Given data :-

i) $w = ?$

ii) $\rho = ?$

iii) $S = ?$

$\because 1\ m^3 = 1000$ litres

1 litre $= 10^{-3}\ m^3$

$V = 1$ litre $\Rightarrow 10^{-3}\ m^3$

$W = 7\ N$

Sol: i) $\because$ Specific weight & Weight density :-

$$\boxed{w = \frac{W}{V}}$$

$$\therefore \quad w = \frac{7}{10^{-3}} \Rightarrow w = 7000\ \frac{N}{m^3}.$$

(ii) Mass Density is given by :

$$\boxed{\rho = \frac{w}{g}} = \frac{7000}{9.81} \Rightarrow \rho = 713.5\ kg/m^3.$$

(iii) Sp. gravity of liquid (S) $= \dfrac{\text{Weight density of liquid}}{\text{weight density of water}}$

$$S = \frac{7000}{9.81 \times 1000} = 0.7135$$

Lighter than water because $S < 1$.

Given liquid

If $S < 1$ Lighter than water

If $S = 1$ water

If $S > 1$ Heavier than water.

Q. Calculate density, Specific weight & weight of one litre of Petrol of specific gravity $= 0.7$.

Given data:- i) $\rho = ?$ $\quad 1\,m^3 = 1000$ litres

ii) $w = ?$ $\quad \therefore 1$ litre $= 10^{-3}\,m^3$

iii) $W = ?$

$V = 1$ litre $\Rightarrow 10^{-3}\,m^3$

$S = 0.7$.

$$\left[\because S_{liquid} = \frac{\rho_{liquid}}{\rho_{water}}\right]$$

Sol^n: i) $\because$ Density is given by:

$$\rho_{liquid} = S_{liquid} \times \rho_{water}$$

$$\therefore \rho_{liquid} = 0.7 \times 1000$$

$$\therefore \rho_{liquid} = 700\ kg/m^3$$

$$\left[\because S_{liquid} = \frac{w_{liquid}}{w_{water}}\right]$$

ii) weight density of the liquid:-

$$w_{liquid} = S_{liquid} \times w_{water}$$

$$\therefore w_{liquid} = 0.7 \times 9810$$

$$\therefore w_{liquid} = 6867\ N/m^3$$

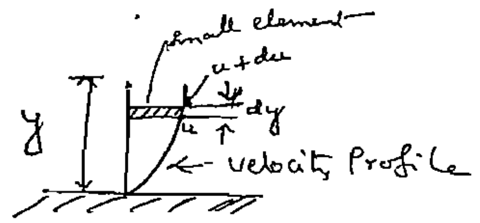
iii) Weight density $(w) = \dfrac{\text{Weight } (W)}{\text{Volume } (V)}$

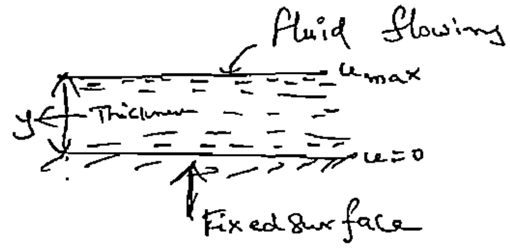
$$\therefore W = w \times V$$

$$= 6867 \times 10^{-3}$$

$$= 6.87\,N$$

* Viscocity (Dynamic Viscocity) :- It is defined as the resistance offered to a layer of fluid when it moves over another layer of fluid. (u = velocity)





∴ Rate of shear stress is directly proportional to the velocity gradient.

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\therefore \quad \tau = \mu \frac{du}{dy}}$$

where, μ = Coefficient of $\frac{du}{dy}$ (Constant of proportionality)

μ = Dynamic viscocity

$$\boxed{\therefore \mu = \frac{\tau}{\frac{du}{dy}}}$$

⇒ It's unit is $\frac{Ns}{m^2}$

⇒ 1 Poise $= \frac{1}{10} \frac{Ns}{m^2}$

* kinematic Viscocity :- It is defined as the ratio of dynamic Viscocity to the density (mass density) of a Fluid

Mathematically, kinematic viscocity (ν) = $\frac{\text{Dynamic Viscocity } (\mu)}{\text{Density } (\rho)}$

$$\boxed{\therefore \nu = \frac{\mu}{\rho}}$$

⇒ It's unit is m^2/s

⇒ 1 stoke $= 10^{-4}\, m^2/s$

* Newton's law of Viscocity:

It states that, "The shear stress in a flowing fluid is directly propotional to the rate of shear strain."

Mathematically,

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\therefore \tau = \mu \cdot \frac{du}{dy}}$$

where,
μ = viscocity (or) Dynamic Viscocity.

→ Significance: The fluids which follow Newton's law of Viscos are called "Newtonian fluids".

* Types of Fluids:

→ Fluids are classified into 5 types.

1. Ideal fluid
2. Real fluid
3. Newtonian fluid
4. Non Newtonian fluid
5. Ideal Plastic Fluid

1. Ideal Fluid:- (Imaginary fluids) A fluid which is incompressible & in which viscocity is zero then, it is called as an Ideal fluid.

2. Real Fluid:- A fluid which is incompressible & in which viscocity is present then, it is called as a Real fluid.

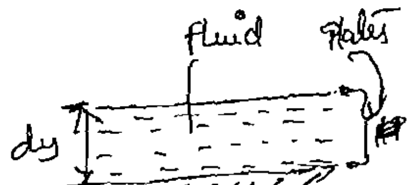
3) <u>Newtonian Fluid</u> : A fluid in which shear-stress i directly proportional to the velocity gradient is cal "Newtonian fluid".

4) <u>Non Newtonian Fluid</u>:- A fluid in which shear stres is not proportional to the rate of shear strain is called "Non-Newtonian fluid."

5) <u>Ideal-Plastic fluid</u>:- A fluid in which shear stress is more than the yield value & in which shear stress is directly proportional to shear strain, is called as "Ideal Plastic Fluid".

* <u>Problem to determine fluid viscocity between two plates</u>

Q) A plate 0.025 mm distant from a fixed plate, moves at 60 cm/s & requires a force of 2 N per uni area to maintain this speed. Determine fluid viscoci between the plates.



<u>Given Data</u>:

$dy = 0.025\,mm$

$= 0.025 \times 10^{-3}\,m$

$u = 60\,cm/s \Rightarrow 60 \times 10^{-2}\,m/s$

Force of 2 N per unit area $= 2\frac{N}{m^2} = \tau = \frac{2N}{m^2}$, $\mu = ?$

Sol: $\because$ Viscosity of fluid, $du = 60\times10^{-2}$ m/s

$\therefore$ Distance between the plates, $dy = 0.025\times10^{-3}$ m

Also, Shear stress is given by:-

$$\boxed{\therefore\ \tau = \mu \cdot \frac{du}{dy}}$$

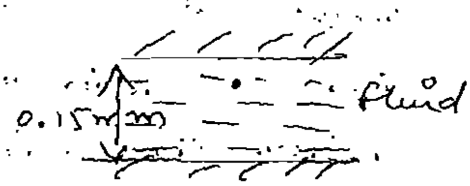
$$2 = \mu \times \frac{60\times10^{-2}}{0.025\times10^{-3}}$$

$$\therefore \mu = \frac{2\times 0.025\times10^{-3}}{60\times10^{-2}} = \underline{8.33\times10^{-3}}\ \frac{Ns}{m^2}$$

Q) A flat plate of area 1.5×10^{6} mm² is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force & power required to maintain this speed, if the fluid separating them is having viscosity as 1 Poise.

Given Data: $A = 1.5\times10^{6}$ mm²

$= 1.5$ m²



$du = 0.4$ m/s

$dy = 0.15$ mm or 0.15×10^{-3} m

i) $F = ?$ & ii) $P = ?$

$\mu = \frac{1}{10}$ Ns/m²

Sol:- $\because$ shear stress is given by:-

$$\boxed{\tau = \mu \cdot \frac{du}{dy}}$$ from Newton's law of Viscosity

$$\therefore \tau = \frac{1}{10} \cdot \frac{0.4}{0.15 \times 10^{-3}}$$

$$\therefore \tau = 266.67 \; N/m^2$$

Also $\tau = \frac{F}{A} \Rightarrow F = \tau \cdot A$

$$= 266.67 \times 1.5$$

Ans:- (i) $F = 400 \; N$

Now, $\because$ Power required is given by:-

power

$$\boxed{P = F \times du}$$

$$= 400 \times 0.4$$

$$P = 160 \; N \cdot m/s$$

$N m - J$

$N \frac{m}{s} \Rightarrow \frac{J}{s} = \text{watt}.$

Ans (ii). $\therefore P = 160$ watts

UNIT - II

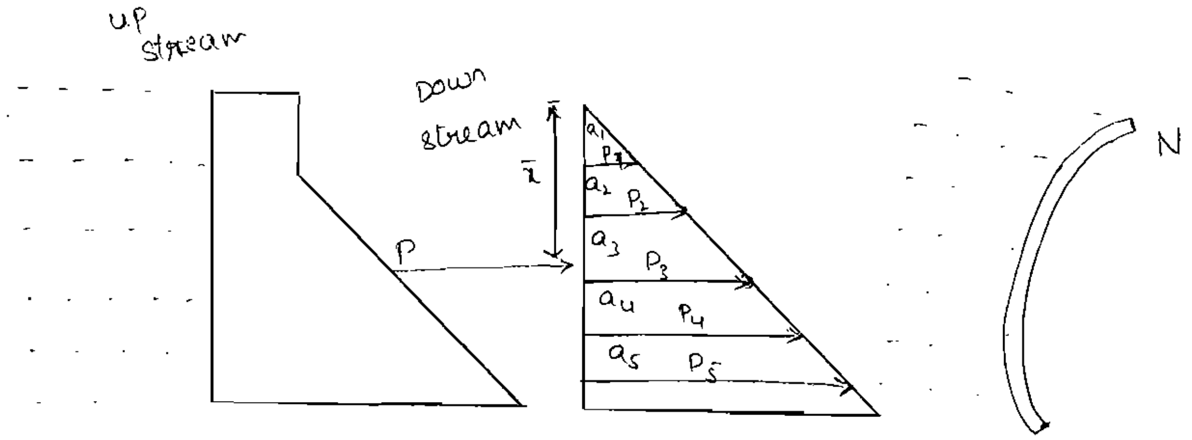
Fluid statics

NO It consists of a thin metallic tube having deep circumferential curvations called "Bellows". As the pressure changes these Bellows expands/ contracts and cause movements of pointer on a graduated circular disk as shown in figure NO

* **Hydrostatic forces on surfaces:-**

Total pressure $= P = P_1 a_1 + P_2 a_2 + P_3 a_3 + \cdots + P_n a_n$

$\bar{x}$ = centre of pressure.



Hydrostatic force

The pressure entered by the static fluid on the surfaces with which it comes into contacts is known as Hydrostatic pressure.

when fluid is at rest condition velocity gradiant and shear stress (τ) is equal to zero.

Total pressure:

It is the sum of the intensite of pressure on different strips of Surface entered by the static fluid.

$$P = P_1 a_1 + P_2 a_2 + P_3 a_3 + \cdots + P_n a_n$$

where, P is known as Total pressure.

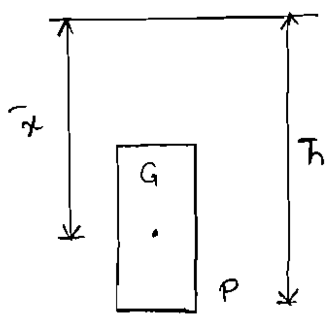
$P_1 a_1$ is the P. intensities of pressure.

Centre of pressure:

It is defined as the point through which the total pressure is assumed to act is known as centre of pressure.

(or)

The point of application of total pressure is known as centre of pressure. It is denoted by $\bar{h}$.



* Types of immerged bodies:-

There are mainly 4 types of immersed bodies

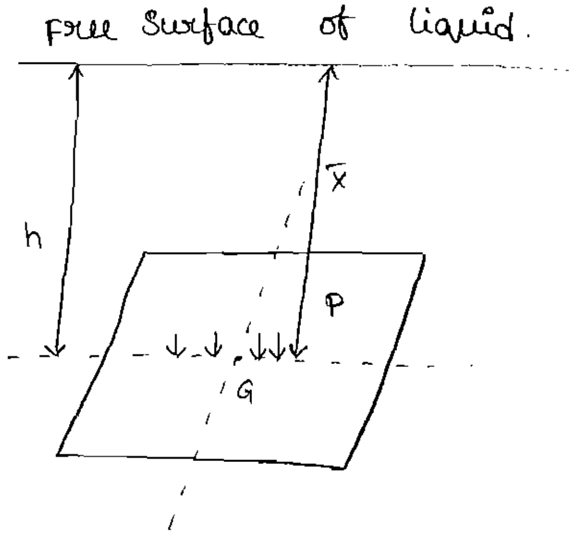
They are i, Horizontally immersed plane surface.

ii, Vertically immersed plane surface.

iii, Inclined immersed plane surface.

iv, Curved surfaces → (or) Submerged

1, Horizontally immersed plane Surfaces:-



consider a plane horizontal surface immersed in a static fluid. As every point of the surface is at the same depth from the free surface of the liquid the pressure intensity will be equal on the entire surface and is equal to

$$p = \rho g h$$

$$p = \omega \cdot h$$

Total pressure is

$$P = p_1 a_1 + p_2 a_2 + \dots$$

$$P = \omega \cdot h \times A$$

$$P = \omega \cdot A \times \bar{x}$$

Centre of pressure:

$$h = \bar{x}$$

Since, every point on the surface of the body is at equal distance from the free surface of the liquid. Hence, the intensity of pressure on the surface is constant.

Hence $\bar{x} = h$ $(\because h = \bar{h})$

(The figure shows a tank.

a, Total pressure on the bottom of the tank.

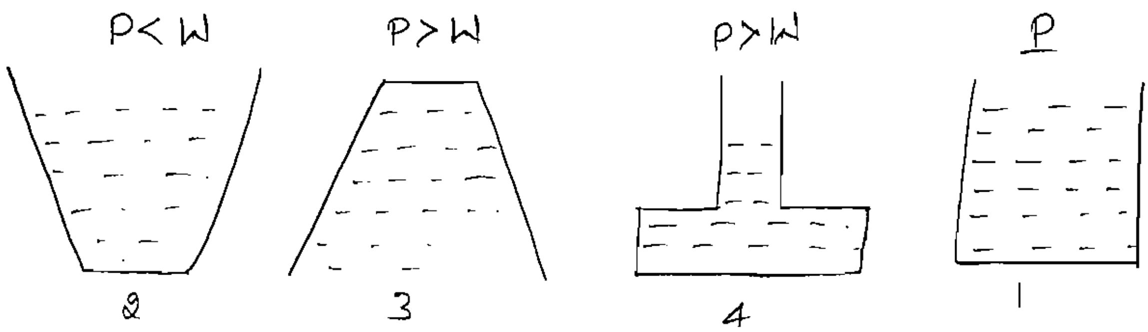
b, wt. of water in the tank.)

* The figure shows a tank.

a, Total pressure on the bottom of the tank

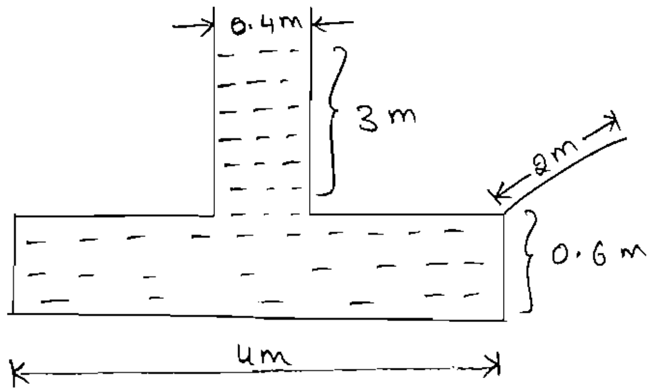
b, wt. of water in the tank.

c, Hydrostatic paradox b/n results ① & ② if width of the tank is 2m.



Hydrostatic paradox:-

The apparent contridiction in the hydrostatic force on the container base and weight of the liquid inside the container is known as hydrostatic paradox.



a, $P = \overset{w \cdot h}{(\rho \cdot g \cdot h)} \times A$

$= 1000 \times 9.81 \times (3+0.6)\ (4\times2)$

$= 282.528$ N

$= 282.52$ kN

b, Weight of the fluid contained in the container $= w \times V$

$= (1000 \times 9.81) \times [3 \times 0.4 \times 2 + 4 \times 2 \times 0.6]$

$= 70632$ N

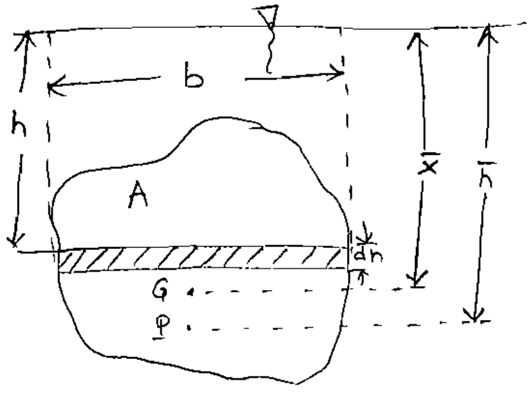
$= 70.632$ kN

c, Hydrostatic paradox

$= (282.52 - 70.632)$ kN

$= 211.88$ kN

* vertical plane surface submerged in liquid:



consider a plane vertical surface immersed in a liquid as shown in figure.

Let

A = Total area of the surface.

$\bar{h}$ = distance of centre of pressure from the free surface of the liquid.

$\bar{x}$ = distance of centroid of the area from the free surface of the liquid

G = ~~(cent)~~ centroid of plane surface.

P = centre of pressure.

B = width of the elementary strip

dh = depth of the elementary strip

h = centroid of the elementary strip

consider a strip of thickness dh and width b at a depth h from the free surface of the liquid.

$$\underline{P} = P_1 a_1 + P_2 a_2 + P_3 a_3 + \cdots + P_n a_n$$

$$\underline{P} = \omega \cdot h$$

$$\underline{P} = \rho \cdot g \cdot h \qquad [\underset{\text{strip area.}}{dA} = b \cdot dh \;\&\; \int dA \cdot h = \bar{x}]$$

$$dF = \underbrace{(\rho \cdot g \cdot h)}_{p} \cdot \underbrace{dA}_{A}$$

$$\int dF = \int \rho . g . h . b . dh$$

$$F = \rho g . \int h . b . dh$$

$$F = \rho g . \int h . dA$$

$$F = \rho . g . \int dA . h$$

$F = \rho . g \times \int$ Area of Surface $\times$ Distance of centroid of elementary strip

$$\int dA = A$$

$$\int h = \bar{x}$$

$\therefore$ Total pressure force acting on the liquid is

$$F = \rho . g . A . \bar{x}$$

Total pressure

$$\boxed{F = \omega . A . \bar{x}}$$

* Centre of pressure:-

centre of pressure is obtained by using varignon's theorem.

Moment of resultant force = Algebric sum of moment of its components.

$$F \times \bar{h} = dF \times h$$

$$dF = \rho . g . h . dA$$

$$dF = \rho.g.h.\ b.dh$$

$$F \times \bar{h} = \int (\rho.g.h.b.dh)\, h$$

$$F \times \bar{h} = \int \rho.\ g.\ h^2.\ b.dh$$

$$F \times \bar{h} = \rho g \int dA.h^2$$

$$\therefore\ F \times \bar{h} = \rho.g.I_0$$

$$I_0 = I_G + A.\bar{x}^2$$

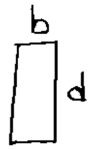
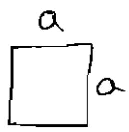
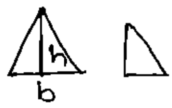
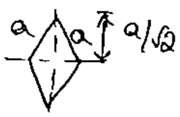
$$F \times \bar{h} = \rho.g.\ (I_G + Ax^2)$$

$$\rho.g.A\bar{x}\,\bar{h} = \rho.g\ (I_G + A.\bar{x}^2)$$

$$\therefore\ \bar{h} = \frac{\cancel{\rho.g}\ (I_G + A.\bar{x}^2)}{\cancel{\rho.g}\ A.\bar{x}}$$

$$\bar{h} = \frac{I_G + A\bar{x}^2}{A.\bar{x}}$$

$$\boxed{\therefore\ \bar{h} = \frac{I_G}{A.\bar{x}} + \bar{x}}$$

Sl. No.	Shape	Area	Centroid from top	I_{xx}
1.		bd	$d/2$	$\frac{bd^3}{12}$
2.		a^2	$a/2$	$a^4/12$
3.		$\frac{1}{2}bh$	$\frac{2h}{3}$	$\frac{bh^3}{36}$
4.		$\frac{1}{2}bh$	$h/3$	$\frac{bh^3}{36}$
5		$\frac{\pi D^2}{4}$	$\frac{D}{2}$	$\frac{\pi D^4}{64}$
6	 equilateral Δ^{le}	$\frac{\sqrt{3}}{4}a^2$	$\frac{2h}{3}$ (where $h = \frac{\sqrt{3}}{2}a$)	$\frac{bh^3}{36}$
7		a^2	$\frac{a}{\sqrt{2}}$	$\frac{a^4}{12}$
8		$\frac{\pi(D^2-d^2)}{4}$	$\frac{D}{2}$	$\frac{\pi(D^4-d^4)}{64}$

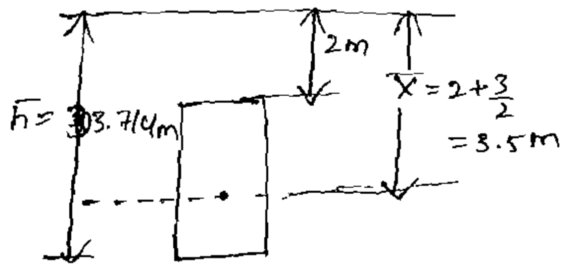
For vertically immersed bodies:-

$$\boxed{\begin{aligned} P &= w \cdot A \cdot \bar{x} \\ \bar{h} &= \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} \end{aligned}}$$

Problems :-

1) calculate the Total pressure and center of pressure on a velocity immersed Rectangular body 2m×3m immersed such that the top is at a depth of 2m from the free surface of a liquid specific wt of water = 10 kN/m³

sol



Area $= 2 \times 3 = 6m^2$

$\bar{x} = 2 + \frac{3}{2} = 3.5m$

$I_{xx} = \frac{bd^3}{12} = \frac{2 \times 3^3}{12} = \frac{\cancel{3}\cdot 3\cdot 3}{\cancel{6}_2}$

$= 4.5 m^4$

$P = W.A \times \bar{x}$

$= 10 \times 6 \times 3.5$

$= 210$ kN

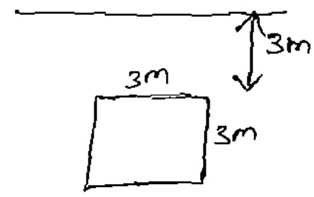
$\bar{h} = \bar{x} + \frac{I_{xx}}{A.\bar{x}}$

$= 3.5 + \frac{4.5}{6(3.5)}$

$\therefore \bar{h} = 3.714m$ from the free surface of the liquid

2) A square body of size 3m×3m is immersed vertically such that the top is at a depth of 3m from the free surface of liquid specific wt of water = 10kN/m³

sol



Area $= 9m^2$

$\bar{x} = 3 + \frac{3}{2} = 4.5m$

$I_{xx} = \frac{3^4}{12} = 6.75m^4$

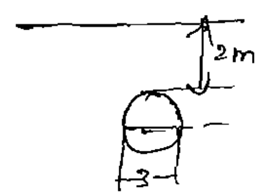
$P = w.A.\bar{x} = 10 \times 9 \times 4.5 = 405$ kN

$\bar{h} = \bar{x} + \frac{I_{xx}}{A.\bar{x}} = 4.5 + \frac{6.75}{9 \times 4.5} = 4.66m$

$= 4.66m$ from the free surface of the liquid

3) A circular body 3m in diameter is immersed vertically such that the top is at a depth of 2m from the free surface of the liquid calculate the total pressure & center of pressure.

sol



Area $= \frac{\pi D^2}{4} = \frac{\pi \times 3^2}{4}$

$= 7.06m^2$

$\bar{x} = 2 + \frac{3}{2} = 3.5m$

$I_{xx} = \frac{\pi D^4}{64} = 3.97m^4$

$P = 10 \times 7.06 \times 3.5$

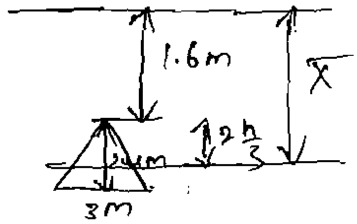
$= 247.38$ kN

$\bar{h} = \bar{x} + \frac{I_{xx}}{A.\bar{x}} = 3.5 + \frac{3.97}{7.06 \times 3.5}$

$= 3.66m$ from the free surface of the liquid

4) A triangle body of base width 3m and height 2.4m is immersed vertically such that the verten is at a depth of 1.6m from the free surface of a liquid specific wt of water = 10kN/m

Sol



$$\text{Area} = \frac{1}{2} \times 3 \times 2.4$$

$$= 3.6 m^2$$

$$\bar{x} = 1.6 + \frac{2}{3} \times 2.4 = 3.2 m$$

$$I_{xx} = \frac{bh^3}{36} = \frac{3 \times (2.4)^3}{36} = 1.152 m^4$$

$$P = w \cdot A\bar{x} = 10 \times 3.6 \times 3.2 = 115.2 kN$$

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 3.2 + \frac{1.152}{3.6 \times 3.2}$$

= 3.3m from the free surface of the liquid

5) A triangular body of base width 3m & ht 2.4m is immersed vertically such that the base is at a depth of 1.6m from the free surface of the liquid spe wt of H_2O = 10kN/m

$$\text{Area} = 3.6 m^2$$

$$\bar{x} = 1.6 + \frac{\cancel{2.4}^{0.8}}{3} = 2.4 m$$

$$I_{xx} = \frac{bh^3}{36} = 1.152 m^4$$

$$P = w \cdot A \cdot \bar{x} \quad \left[\because \frac{\cancel{3} \times (2.4)^3}{\cancel{36}_{12}} \right]$$

$$= 10 \cdot 3.6 \cdot 2.4$$

$$= 86.4 kN$$

$$h = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}} = 2.4 + \frac{1.152}{3.6 \times 2.4} = 2.53 m$$

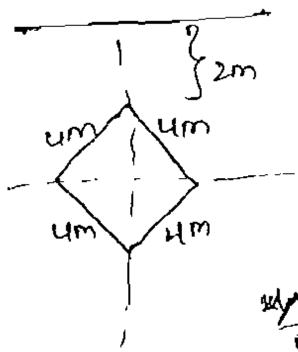
= 2.53 m from the free surface of the liquid

6) A square body of side 4m x 4m is immersed such that one of the diagonal is ||el to free surface & top is at a depth of 2m from the free surface Find.

(a) Total pressure

(b) Centre of pressure

sol



Area = 16 m²

$\bar{x} = 2 + \frac{4}{\sqrt{2}}$

$= 4.83 m$

$I_{xx} = \frac{a^4}{12} = \frac{4.4.4.4}{12} = 21.33 m^4$

$4^2 = 2x^2$

$x^2 = \frac{16}{2} = 1$

$x = 2.828$

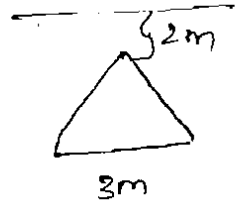
$P = \omega.A.\bar{x}$

$= 10 . 16 . 4.83 = 772.8 kN$

$\bar{h} = \bar{x} + \frac{I_{xx}}{A.\bar{x}} = 4.83 + \frac{21.33}{16.(4.83)}$

$= 5.106$ m from the free surface of the liquid

(7) An equilateral Δle of sides 3m is immersed vertically such that the top is at a depth of 2m from the free surface of the liquid calculate the total pressure & centre of pressure Assumed spe wt of H_2O = 10kN



$h = \frac{\sqrt{3}}{2}.a$

Area $= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} . 9$

$= 3.89 m^2$

$$\bar{x} = 2 + \frac{2h}{3}$$

$$= 2 + \frac{2}{3}\cdot\frac{\sqrt{3}}{2}\cdot a$$

$$= 2 + \sqrt{3}$$

$$= 2 + 1.732$$

$$= 3.732\ m$$

$$I_{xx} = \frac{bh^3}{36}$$

$$= \frac{\cancel{3}\times\left(\frac{\sqrt{3}}{2}\right)^3 a^3}{\cancel{36}\ 12} = \frac{\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}\ 9\cdot 3}{12}$$

$$= \frac{17.5}{12} = 1.45\ m^4$$

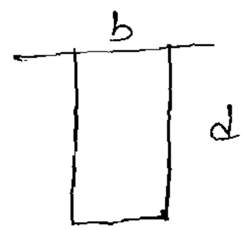
$$P = \omega\cdot A\cdot\bar{x} = 10\cdot(3.89)(3.732)$$

$$= 145.09\ kN$$

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A\cdot\bar{x}} = 3.732 + \frac{1.45}{3.89(3.732)}$$

$= 3.829\ m$ from the free surface of the liquid.

8) A rectanguler body of breadth b and depth d is immersed vertically such that the top coincides with the free surface of the liquid Find the position of centre of pressure



$$\bar{h} = \bar{x} + \frac{I_{xx}}{A\cdot\bar{x}}$$

$$= \frac{d}{2} + \frac{bd^3}{\cancel{12}\ 6\cdot b\cdot d\cdot\frac{d}{\cancel{2}}}$$

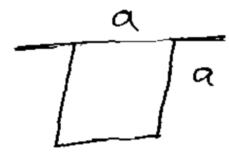
$$= \frac{d}{2} + \frac{d}{6}$$

$$= \frac{d}{2}\left[1 + \frac{1}{3}\right]$$

$$\bar{h} = \frac{d}{2} \cdot \frac{4}{3} = \frac{2d}{3}$$

9) A square body of size a x a is immersed vertically such that the top coincides with the free water surface locate the centre of pressure

<u>Sol</u>

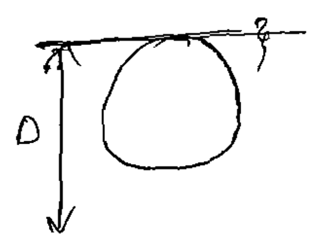


$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$

$$= \frac{a}{2} + \frac{a^4}{\cancel{12} \cdot a^2 \cdot a / \cancel{2}}_{6}$$

$$= \frac{a}{2} + \frac{a}{6} = \frac{2a}{3}$$

10) A circular body of diameter "D" is immersed such that the top coincides with the free water surface locate the position of centre of pressure



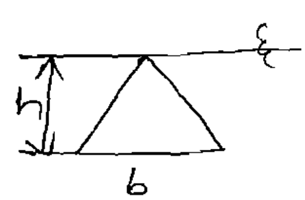
$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$

$$= \frac{D}{2} + \frac{\frac{\pi D}{64}}{\frac{\pi D}{4} \times \frac{D}{2}} \qquad = \frac{D}{2} + \frac{D}{\cancel{64}_{8}} \cancel{\times 8}$$

$$= D\left[\frac{4+1}{8}\right]$$

$$= \frac{5D}{8}$$

11) A triangular body base width b and height h is immersed vertically such that the open coincides with the free surface of the liquid calculate the centre of pressure



$$\bar{h} = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$

$$= \frac{2}{3} \ldots \frac{\frac{bh^3}{36}}{\frac{1}{2} \times b \times 2 \times h}$$

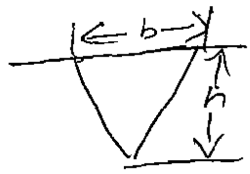
$$= \frac{2}{3}h + \frac{\frac{bh^3}{36}}{2} \times \frac{3}{bh^2}$$

$$= \frac{2}{3}h + \frac{h}{12}$$

$$= h\left[\frac{2}{3} + \frac{1}{12}\right] = h\left[\frac{8+1}{12}\right]$$

$$= h \cdot \frac{9}{12} = \frac{3h}{4}$$

12) A triangle having base b and height h is immerses such that the base coincides with the free surface of the liquid locate the centre of pressure



$$h = \bar{x} + \frac{I_{xx}}{A \cdot \bar{x}}$$

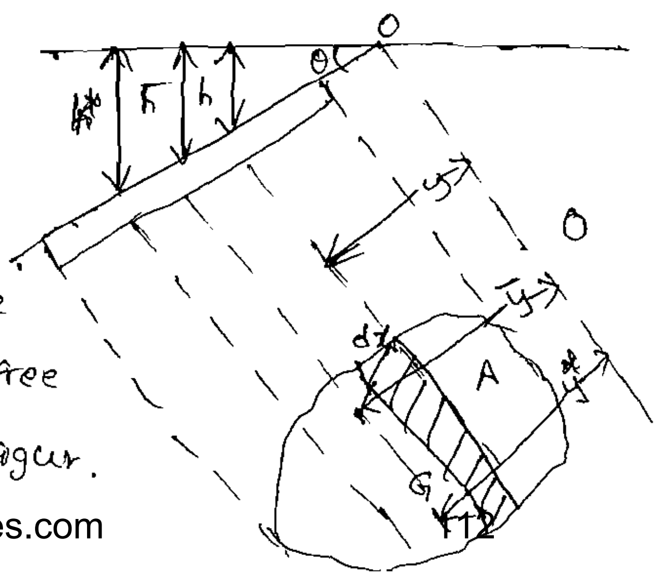
$$= \frac{1}{3} \cdot h + \frac{\frac{bh^3}{36}}{\frac{1}{2} \cdot bh \cdot \frac{h}{3}}$$

$$= \frac{1}{3}h + \frac{h}{36} \times 6$$

$$= \frac{1}{3}\left[h + \frac{h}{2}\right] = \frac{h}{2}$$

Inclined surface submerged in liquid:-

prove that centre of pressure always lies below centroid of the inclined surface.



Consider a plane surface of orbitary shape immersed in liquid in such a way that the plane of the surface makes an angle θ with the free surface of the liquid as shown in figur.

Let 'A' total area of Inclined Surface h = depth of small elementary strip from the free surface of the liquid the $\bar{h}$ = depth of centroid of inclined area from the free surface h^* = Distance of centre of pressure from the surface of the liquid θ = Angle made by the plane of the surface with the free surface of the liquid

Let the plane surface of produced meet the free liquid surface at O then $\overline{OO}$ is the axes ⊥r to the plane of the surface

Let y = Distances of the elementary strip from the axes $\overline{OO}$ $\bar{y}$ = Distances of centroid of the inclined surface from the axes OO y^* = Distance of centre of pressure from the axes $\overline{OO}$

Consider a small elementary strip of area dA pressure force acting on the elementary strip.

Let

dF = pressure force acting on the elementary strip

$dF = \int dF$

$dF = P \times \text{Area} = (\rho \cdot g \cdot h) \times dA$

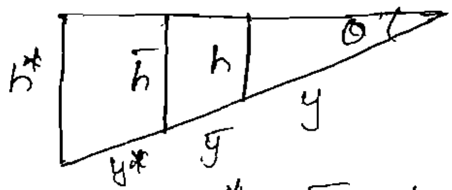
$= \rho \cdot g \cdot h \times b \cdot dy$

$dF = \rho \cdot g \cdot h \times b \cdot dy$

$= \rho \cdot g \times b \cdot y \sin\theta \, dy$

[∵ Varigonon's Theorem

$A\bar{y} = \int dA \cdot h$



$\sin\theta = \frac{h^*}{y^*} = \frac{\bar{h}}{\bar{y}} = \frac{h}{y}$

$= \rho \cdot g \cdot \sin\theta \int b \cdot dy \cdot y$

$= \rho . g \sin\theta \ A . \bar{y}$ $\quad [\because \bar{h} = \bar{y} \sin\theta]$

$= \rho . g \ A \bar{y} \ \sin\theta = \rho . g . A \bar{h}$

$F = (\text{specific wt} = \omega) \ \omega . A . h$

$\boxed{F = \omega . \bar{h} . A}$ Total pressure acting on the body

$F \times h^{*} = \int dF . y$

$\left[\because \int y^2 dA = \text{moment of inertia}\right.$

$I_O = I_{xx} + A\bar{x}^2$

$\left. I_O = I_{xx} + A\bar{h}^2 \right]$

$F \times y^{*} = \int dF . y$

$= \int \rho . g . h . dA . y$

$= \int \rho g . y \sin\theta \ dA . y$

$= \rho . g \sin\theta \int dA \ y^2$

$= \rho g \sin\theta \ I_O$

$= \rho . g . \sin\theta \ (I_{xx} + A\bar{y}^2)$

$= \rho . g \sin\theta \ (I_{xx} + A\bar{y}^2)$

$F y^{*} = \rho g \sin\theta \ (I_{xx} + A\bar{y}^2)$

$$y^{*} = \frac{\rho g \sin\theta \ (I_{xx} + A\bar{y}^2)}{\rho g \sin\theta \ A\bar{y}}$$

$$y^{*} = \frac{I_{xx} + A\bar{y}^2}{A\bar{y}}$$

$$\boxed{y^{*} = \bar{Y} + \frac{I_{xx}}{A\bar{Y}}}$$

$$y^{*} = \frac{h^{*}}{\sin\theta} = \bar{y} + \frac{I_{xx}}{A\bar{y}}$$

$$\Rightarrow \frac{h^*}{\sin\theta} = \frac{\bar{h}}{\sin\theta} + \frac{I_{xx}}{A\frac{\bar{h}}{\sin\theta}}$$

$$= \frac{1}{\sin\theta}\left(\bar{h} + \frac{I_{xx}\sin^2\theta}{A\bar{h}}\right)$$

$$\boxed{h^* = \bar{h} + \frac{I_{xx}\sin^2\theta}{A\bar{h}}}$$

If $\theta = 90^\circ$ the above equation become same as that of equation which is applicable to equation which applicable to ~~ertically~~ vertically submerged surface for inclined surface,

total pressure $F = \omega \cdot A \cdot \bar{h}$

Centre of pressure $h^* = \bar{h} + \frac{I_{xx}\sin^2\theta}{\bar{h}^2}$

$\boxed{\bar{h} = \bar{x}} \rightarrow$ Note

A rectangular body 3m × ~~2~~ 4m is immersed such that the greatest and least depth are 4m and 2m respectively from the free surface of the liquid

Find
1) Total pressure
2) centre of pressure
· Take specific wt of liquid 10 kN/m^3

Sol

Sol

a) Total pressure force

$= 10 \times (4 \times 3) \times 3$

$= 360$ kN

(b) $\bar{h} = \frac{2+4}{2} = 3$

$h^* = 3 + \frac{\frac{3 \times 4^3}{12} \sin 30^0}{3 \times 4 \times 3}$

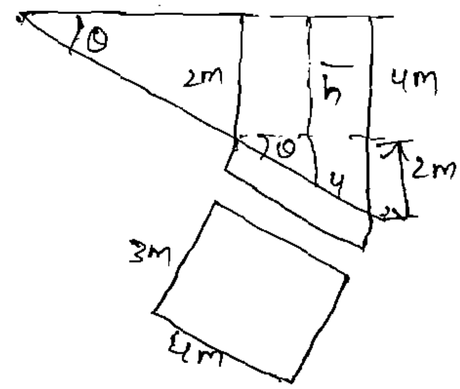
$= 3 + \frac{\frac{3 \times 4^3}{12} \times \left(\frac{1}{2}\right)^2}{3 \times 4 \times 3}$

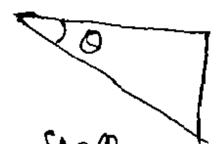
$= 3 + \frac{1}{9}$

$= \frac{27+1}{9}$

$= \frac{28}{9}$

$= 3.11$ m





$\sin\theta = \frac{2}{4} = \frac{1}{2}$

$\theta = 30°$

2) A square body of side 4×4 is immersed such that the plane makes an a angle of 30° with the free surface and top is at the depth 2m from the free surface

Find

(a) Total pressure

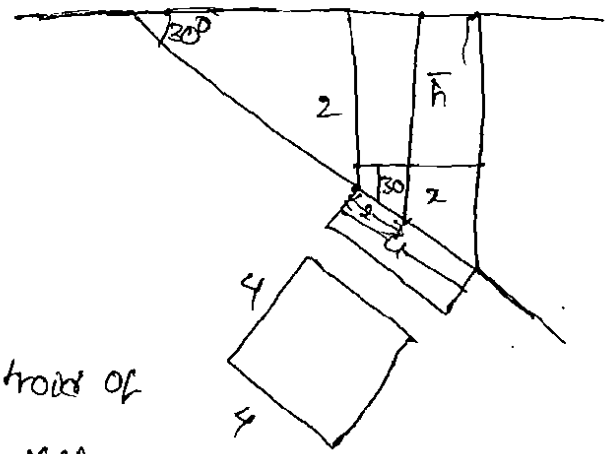
(b) centre of pressure

Sol

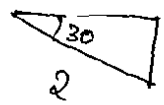
Let $\bar{h} = 2 + x$.

$= 2 + (2$

$= 2(1 + \sin 30^\circ)$



$\bar{h}$ = Initial depth centroid of the inclined surface $x \sin\theta$



$\sin 30 = \frac{x}{2}$

$2 \sin 30 = x$

$\bar{h} = 2(1 + \sin 30^\circ) = 3$

$F = 10 \times (4 \times 4) = 3$

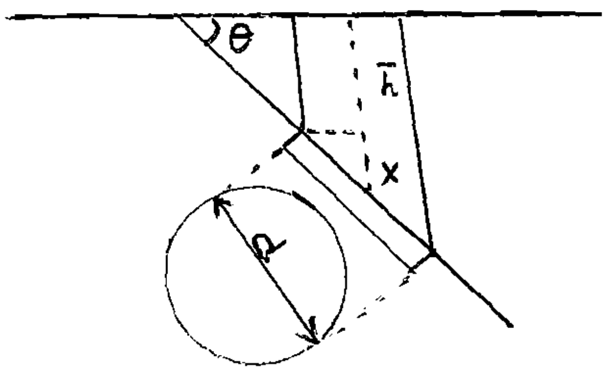
$= 480$ kN

$$h^* = \frac{3 + \frac{4^4}{12} \sin^2 30}{4 \times 4 \times 3}$$

$= 3.11$

1. A circular body of diameter 4m is immersed such that the greatest & least depths are 3m & 1m. Find Total pressure and Centre of pressure.

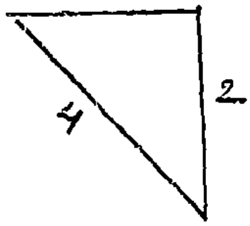
Sol: $\bar{h} = \frac{1+3}{2} = 2m$



$F = 10 \times \pi \times \frac{4^2}{4} \times 2$

$= 251.327 \text{ KN}$

$$\bar{h} = \frac{2 + \frac{\pi \times 4^4}{64} \sin^2\theta}{\pi \times \frac{4^2}{4} \times 2}$$



$= 2 + 16/32 \times \frac{1}{4} \sin^2\theta \quad = 1/4$

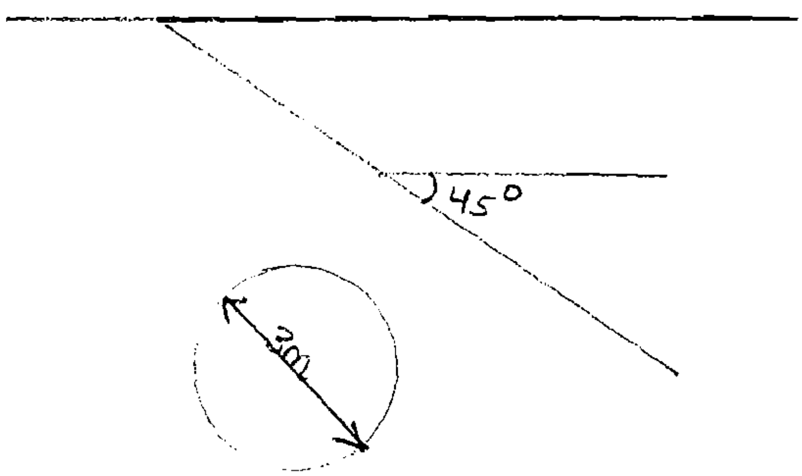
$= 2 + \frac{1}{8} = \frac{17}{8} = 2.125 m.$

$\sin\theta = 2/4.$

$\sin\theta = 1/2.$

2. A circular 3m in diameter is immersed such that the top coincides with free surface of two liquid & make an angle 45° with free surface find total pressure Centre of Pressure?

Sol: $F = 10 \times \frac{\pi \times 3^2}{4} \times \frac{3}{2\sqrt{2}}$



$= 74.97 \text{KN}.$

$\bar{h} = 0 + \frac{3}{2} \times \sin 45^\circ$

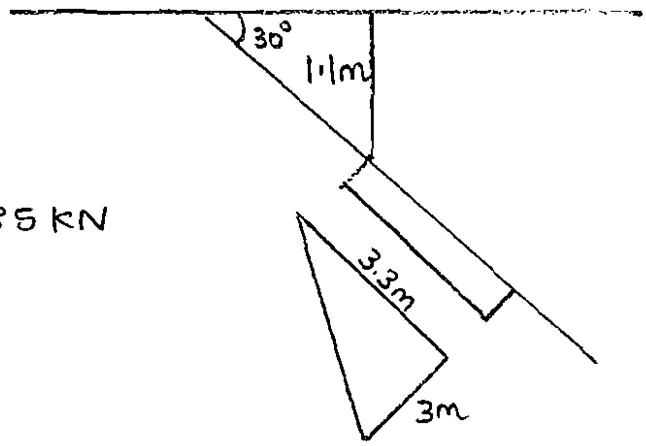
$= \frac{3}{2\sqrt{2}} = 1.06 m.$

$$h^* = \frac{\frac{3}{2\sqrt{2}} + \frac{\pi \times 3^4}{64}\sin^2 45^\circ}{\frac{\pi \times 3^2}{4} \times \frac{3}{2\sqrt{2}}}$$

$$h^* = \frac{3}{2\sqrt{2}} + \frac{\frac{3}{8} \times \frac{1}{2}}{1/\sqrt{2}} = 1.325m$$

$$= 1.06 + \frac{3\sqrt{2}}{16} = 1.325m.$$

3. A Triangular body of base width 3m and height 3.3m is immersed such that the Vertex is at a depth 1.2m from the free surface and plane makes an angle 30° with the free surface. Find a a) Total pressure b) Centre of pressure.



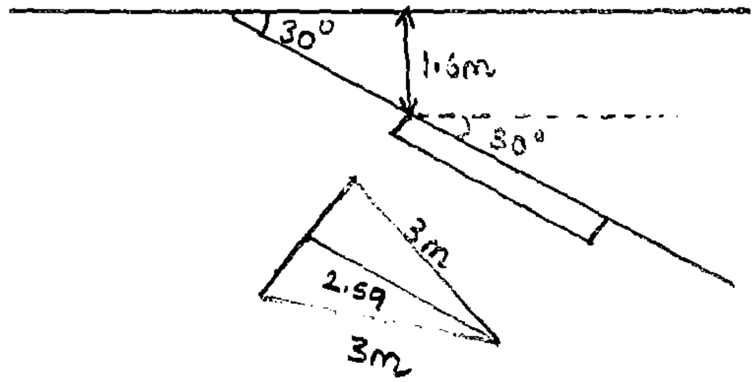
Sol: $\bar{h} = 1.2 + \frac{2 \times 3.3}{3}\sin 30^\circ = 2.3m$

$F = 10 \times \frac{1}{2} \times 3.3 \times 3 \times 2.3 = 113.85\,KN$

$$h^* = 2.3 + \frac{\frac{3 \times 3.3^3}{36} \times \sin^2 30^\circ}{\frac{1}{2} \times 3 \times 3.3 \times 2.3}$$

$= 2.36m$ from the Free Surface of the liquid.

4. An equilateral Δ^{le} 3m×3m is immersed such that the plane makes an angle 30° with the free surface is at depth of 1.6m from the free surface Find Total pressure and Centre of pressure.



Sol: $\bar{h} = 1.6 + \frac{2.59}{3}\sin 30^\circ$

$= 2.03m.$

$A = \frac{\sqrt{3}}{4} \times 3^2 = 3.89m^2.$

$h = \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{2} \times 3 = 2.59m.$

$F = 10 \times 3.897 \times 2.03.$

$= 79.1KN.$

$h^* = 2.03 + \dfrac{\frac{3 \times (2.59)^3}{36} \times \sin^2 30^\circ}{3.897 \times 2.03}$

$= 2.07$ from top.

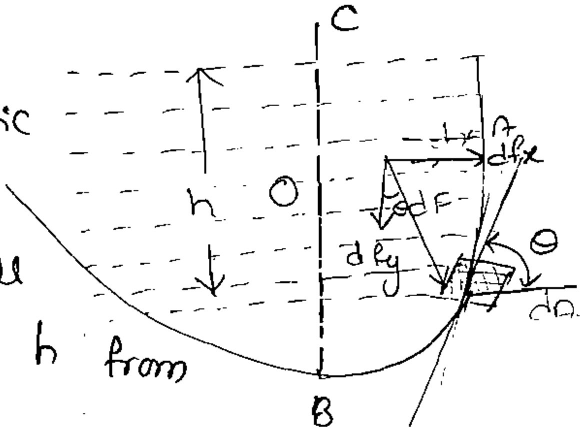
$\sin 30^\circ = \frac{x}{0.86}$

$x = 0.43$

$\bar{h} = 1.6 + 0.43.$

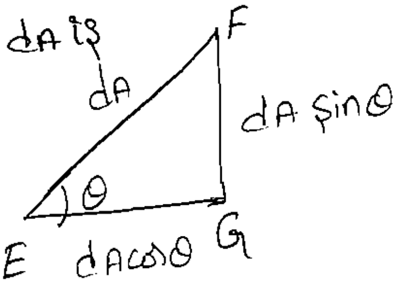
$= 2.03m.$

Curved surface submerged in liquid:-



Let us consider a curved surface AB submerged in static fluid as shown

Let "dA" be the area of small elementry strip at a depth of h from the free water surface.



The pressure intensity on the area dA is

$dp \rightarrow dA = \rho g h.$

pressure force $dF = (\rho . g . h)\, dA$

$$F = \int dF = \int \rho g h \cdot dA.$$

pressure force $dF = (\rho . g . h]\, dA.$

$$F = \int dF = \int \rho g h \; dA.$$

here the direction of forces on the small areas are not in the same direction. It varies from point. Hence integration of equation of curved surface is impossible.

Let resultant force dF on the elementary strip makes an angle θ with the vertical and tangent of the curved surface makes the same angle with the horizontal.

$$dF = \sqrt{dF_x^2 + dF_y^2}$$

$$F = \sqrt{F_x^2 + (F_y)^2}$$

$dF_x = dF \sin\theta = \rho g h \, dA \sin\theta$.

$dF_y = dF \cos\theta = \rho g h \, dA \cos\theta$.

Total force acting on teu curved surface.

$$F_x = \int dF_x = \int \rho g h . dA \sin\theta = \rho . g \int h . dA \sin\theta$$

$$F_y = \int dF_y = \int \rho . g . h \, dA \cos\theta = \rho . g \int h \, dA \cos\theta$$

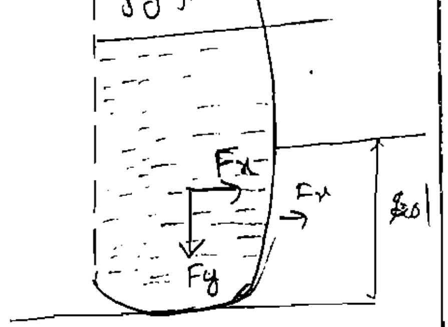
$\rho g \int h . dA \sin\theta$ reprasents teu total pressure force of teu curved surface on teu projected area on teu vertical plane.

$$\rho g \int h . dA \sin\theta \rightarrow F_x = \rho g . A . \bar{x}$$

$$\bar{h} = \bar{x} + \frac{I_{xx}}{A\bar{x}}$$

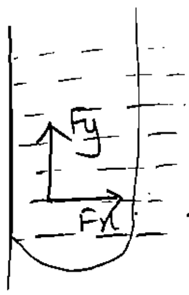
$\rho g \int h \, dA \cos\theta$ reprasents teu wt of teu liquid supported by teu curved surface up to free surface of teu liquid.

$\rho g \int h \, dA \cos\theta \downarrow F_y$



$$\rho g \int h . dA \cos\theta \rightarrow F_y = \omega V.$$

Note :-



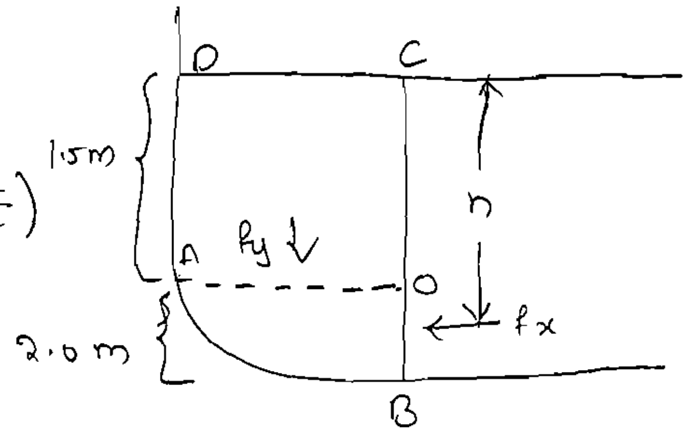
$\rightarrow$ In case figure curved surface AB is not supporting any liquid In such cases F_y = The wt of imaginary Liquid supported by AB upto free surface of teu liquid teu direction F_y will be taken in upward direction

$$F_x = \rho . g . A . \bar{x} = \omega . A . \bar{x}$$

$$h = \bar{x} + \frac{I_{xx}}{A . \bar{x}}$$

$F_y = \omega \times$ volume of teu liquid suported by curved surface.

(1) compute the horizontal and vertical components of the total force acting on a curved surfaces AB which is in the form of quadrant of a circle of radius 2m 2mts as shown in fig. Take the width of the gate as unity.



$F_x = \rho . g . A \bar{x}$

$= \omega . A . \bar{x}$

$= 1000 \times 9.81 \times (2 \times 1) . (1.5 + \frac{2}{2})$

$= 981 \times (2 \times 2.5)$

$= 49050 N$

$= 49.05 kN.$

$\bar{h} = \bar{x} + \frac{I_{xx}}{A . \bar{x}}$

$= 2.5 + \frac{1 \times 2}{(12) 1 \times 2 \times 2.5}$

$= 2.5 + \frac{1}{7.5}$

$= 2.33 m$

$F_y = \omega \times V$

$= 1000 \times 9.81 \times [\underset{OCAO}{(1.5 \times 2.0 \times 1)} + \underset{AOB}{(\frac{\pi \times 2^2}{4} \times 1.0)}$

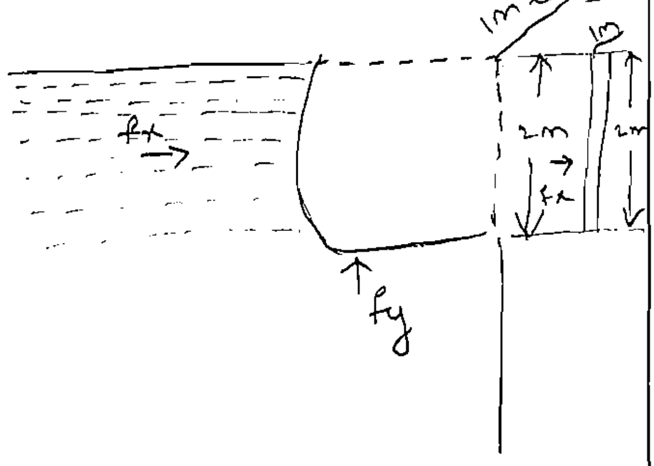
$= 60.249 kN$

$F = \sqrt{F_x^2 + F_y^2}$

$\therefore F = \sqrt{(49.05)^2 + (60.249)^2}$

$\therefore F = 77.69 kN.$

(1) Fig shows a gate having a quadrant shape of radius 2m. find the resultant force due to water per mt length of the dam. Find also the angle at which the total force will act.



sol:-

$F = \sqrt{F_x^2 + F_y^2}$

$\tan\theta = \frac{F_y}{F_x}$

$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]$

$F_x = w \cdot A \cdot \bar{x}$

$= (1000 \times 9.81) \times (2 \times 1) \times \frac{2}{2}$

$= 19.62$ kN.

F_y = wt. of the water supported by the curved surface (Imaginary)

$= w \cdot \bar{V}$

$= w \cdot A \times L$

$= 1000 \times 9.81 \times \frac{\pi \times 2^2}{4} \times 1$

$= 30.8$ kN.

$\therefore F = \sqrt{F_x^2 + F_y^2}$

$= \sqrt{(19.62)^2 + (30.8)^2}$

$= 36.52$ kN.

$\tan\theta = \frac{30.8}{19.62}$

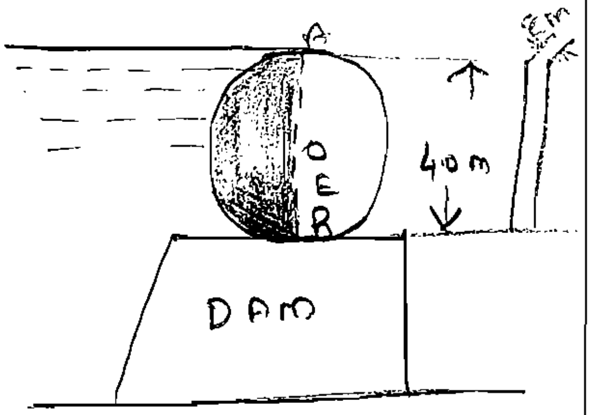
$\therefore \theta = \tan^{-1}\left[\frac{30.8}{19.62}\right]$

$\boxed{\therefore \theta = 57^\circ 30'}$

(2) Find magnitude and direction of the resultant force due to

water acting on a roller rate of cylidrical form 4m diameter when the gate is placed on the dam. In when the gate is paced on the dam. In such a way that water is just go into spiln Take the length of the gate as 8m.

Sol:- $F = \sqrt{F_x^2 + F_y^2}$



$\tan\theta = \frac{F_y}{F_x}$

$\theta = \tan^{-1}\left[\frac{F_y}{F_x}\right]$

$F_x = w \cdot A \cdot \bar{x}$

$= (1000 \times 9.81)(4 \times 8)\frac{4}{2}$

$= 627.8\ kN$

F_y = wt. of water supported by the curved surface (Imaginary)

$= w \times V$

$= (1000 \times 9.81)\frac{\pi \times 2^2}{2} \times 8$

$= 493.10\ kN$

$\therefore F = \sqrt{(627.8)^2 + (493.10)^2}$

$= 798.32\ kN$

$\therefore \theta = \tan^{-1}\frac{493.10}{627.8}$

$\boxed{\therefore \theta = 38^\circ 8'}$

(3) Find the horizontal and vertical component of water pressure acting on the face of a tainter gate of 90° sector of radius 4m as shown in figure. Take the width of the gate as unity.

Sol:- $F = \sqrt{F_x^2 + F_y^2}$

$\theta = \tan^{-1} \frac{F_y}{F_x}$

$\therefore F_x = \omega \cdot A \cdot \bar{x}$

$= (1000 \times 9.81)(5.65 \times 1) \times \frac{5.65}{2}$

$= 156.91$ KN.

$F_y = 1000 \times 9.81 \times 2 \cdot \left.\frac{y^{3/2}}{3/2}\right]_0^{12}$

$= 1000 \times 9.81 \times \frac{4}{3} \; 12^{3/2}$

$= 1000 \times 9.81 \times \frac{4}{3} \sqrt{12} \cdot 12$

$= 1000 \times 9.81 \times 16\sqrt{2}$

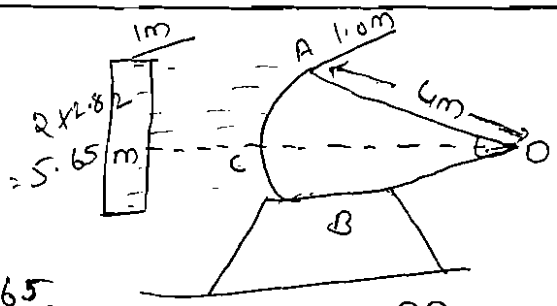
$F_y = 543.725$ kN.

$\therefore F = \sqrt{(706.3)^2 + (543.7)^2}$

$\therefore F = 891.3$ kN.

$\therefore \theta = \tan^{-1}\left[\frac{543.7}{706.3}\right]$

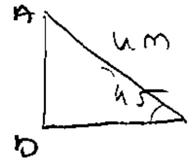
$\boxed{\therefore \theta = 37^\circ 35'}$



$\sin 45 = \frac{AD}{4m}$

$AB = 4 \times \sin 45$

$= 2.82$ m



ND (BUOYANCY

Whenever a body is placed over a liquid either it sinks down or floats on a liquid. If we analyse the phenomena of floating we find that the body placed over a liquid is subjected to the follow two forces

(1) Gravitational force and

(2) up thrust of ~~the~~ the liquid.) ND